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WADC TECHNICAL REPORT NO. 53-36
PART 2

SECURITY INFORMATION

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# EVALUATION OF AIRCRAFT ACCESSORY POWER TRANSMISSION SYSTEMS BY SELECTED ANALYTICAL METHODS


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*JULY 1953*

Unclassified
Ltr, WCLPF, dtd 28 Jan 54
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WRIGHT AIR DEVELOPMENT CENTER

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*July 1953*


*Power Plant Laboratory*
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Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio


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FOREWORD

This report was prepared by the Armour Research Foundation of the Illinois Institute of Technology under USAF Contract No. AF33(038)-21751. The work performed in this program was under the supervision of the Power Plant Laboratory, Directorate of Laboratories, Wright Air Development Center, with Mr. J. D. Delano, Jr. as project engineer, and is covered by RDO No. R-536-232, "Power Plant Power Transmission Systems".

This report is presented in two parts. Part 1 is concerned with the basic assumptions for the various types of aircraft accessory power transmission systems concerned, while this part (Part 2) consists of the derivations of the equations which were utilized in Part 1.

CONTRIBUTING PERSONNEL: R. M. Bergslien, F. E. Bisshopp, S. M. Bolker, J. H. Bonin, H. J. Finison, W. P. Green, R. A. Harmon, L. J. Stratton, O. E. Teichmann, and F. J. Vodvarka.

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# ABSTRACT


A method is presented for evaluating four possible types of aircraft accessory power transmission systems as listed below.

1. Pneumatic
2. Hydraulic
3. Electric
4. Mechanical

This evaluation method is intended for use in deciding which type of accessory transmission system should be used in a given type aircraft.

These systems were analyzed on a minimum weight basis. Weights as well as the increase in fuel weight to compensate for power extracted from the engines were considered on the basis of mission profile and power characteristics of the accessory systems. The minimum weight was mainly determined by the transmission line sizes in the respective systems, while other components were essentially constants in the analysis.

This report is divided into two parts. Part 1 is devoted to listing, for each system, the basic assumptions and required data, and to presenting a step by step procedure for calculating the minimum total weight of each basic system. This part (Part 2) contains derivations of the equations which were utilized in Part 1 without derivation or detailed explanation.

The security classification of the title of this report is "UNCLASSIFIED". While each individual section of this report is not considered classified, the compendium of information contained in this report is considered to be of sufficient importance to require the protection afforded by a RESTRICTED classification.


## PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER

NORMAN C. APPOLD, Colonel, USAF
Chief, Power Plant Laboratory
Directorate of Laboratories

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## SECTION I

## INTRODUCTION

This report is a supplement to Part 1 of Evaluation of Aircraft Accessory Power Transmission Systems by Selected Analytical Methods, WADC Technical Report No. 53-36, which contains the analysis-procedure in concise form for the analysis of the following types of transmission systems:

1. Pneumatic
2. Hydraulic
3. Electric
4. Mechanical

This report contains the derivations of the equations which were used in Part 1 without derivation or detailed explanation.

# ANALYSIS OF THE WEIGHT OF PNEUMATIC POWER TRANSMISSION SYSTEMS

## Section II

### A. Introduction

The derivations of the various major parameters and curves used in the analysis of the pneumatic systems are grouped under three main headings: Straight Bleed System, Bleed and Burn System, and Weight of the Air Turbine. These headings are parallel to those used in Part 1 of this report. The derivations under these headings are presented in the order in which the parameters arise in the analysis procedure.

Some of the parameters and their derivations are the same for both the straight bleed system, and for the bleed and burn system. The head loss parameter, $\beta$, the dimensionless pressure drop parameter, $X^*$, and the specific fuel consumption, $C''_{PX}$, fall in this category and are derived in sections 1, 2, and 6 under the Straight Bleed System.

### B. Nomenclature

| | |
|---|---|
| $A$ | annular area of the rotor, in.$^2$ |
| $a$ | taper |
| $C_{b1}$ | thrust correction factor at cruise conditions |
| $C'_{b1}$ | fuel flow correction factor at cruise conditions |
| $C''_{b1}$ | specific fuel consumption chargeable to the accessories, lb of fuel per hr/lb of bleed air per second |
| $C_u$ | nozzle coefficient |
| $c$ | per cent increase in rotor blade height over nozzle blade height |
| $c_p$ | specific heat at constant pressure Btu/lb °F |
| $D$ | duct diameter or diameter, ft |
| $D^*$ | dimensionless duct parameter |
| $d$ | turbine tip diameter, in. |
| $F_n$ | net engine thrust, lb |
| $f$ | friction factor, factor or function |
| $g$ | acceleration due to gravity, ft/sec$^2$ |

| | |
|---|---|
| $h$ | blade height, in. |
| HP | horsepower |
| $K$ | head loss coefficient or proportionality constant |
| $k$ | ratio of specific heats |
| $\Sigma K$ | total head loss coefficient |
| $L$ | length of duct, ft |
| $N$ | revolutions per minute, 1/min |
| $P$ | absolute pressure, $lb/ft^2$ |
| $R$ | gas constant, ft-lb/lb °R |
| Re | Reynolds number |
| $r$ | ratio or radius, in. |
| $S$ | ratio of fitting weight to weight of duct alone, or pitch, in. |
| $s$ | allowable stress, psi |
| $T$ | taper factor or temperature, °R |
| $t$ | thickness, in. |
| $V$ | velocity, ft/sec |
| $W$ | weight, lb |
| $W_a$ | weight rate of air flow, lb/sec |
| $W_{BL}$ | bleed air flow, lb/sec |
| $W^*_{BL}$ | dimensionless bleed air flow parameter |
| $W_F$ | weight of fuel required to operate accessory system for time, $\tau$, lb |
| $W_f$ | fuel flow for engine, lb/hr |
| $w$ | width, in. |
| $X^*$ | dimensionless pressure drop parameter |
| $\alpha$ | inverse of pressure ratio, $P_{b0}/P_{b1}$ |
| $\beta$ | head loss parameter |

| | |
|---|---|
| $\gamma$ | weight density, $lb/ft^3$ |
| $\zeta$ | aspect ratio |
| $\eta$ | efficiency |
| $\theta$ | admission angle |
| $\lambda$ | blade solidity |
| $\tau$ | duration of power extraction, hr |
| $\Phi$ | horsepower parameter |
| $\psi$ | specific thrust fuel consumption, lb/lb-hr |
| $\omega$ | duct weight coefficient |
| $W_k$ | increment of total aircraft weight due to additional structural requirements and to the fuel required to overcome any increased aerodynamic drag chargeable to the transmission systems, lb |

Subscripts

| | |
|---|---|
| A | area |
| a | air |
| Am | ambient condition |
| BL | bleed air |
| b | blade |
| b1 | conditions at compressor outlet |
| b3 | conditions at burner inlet |
| b4 | conditions at turbine inlet |
| b0 | conditions at turbine discharge |
| C | casing |
| c | cruise |
| cr | critical |
| D | duct |
| d | disk |

| | |
|---|---|
| E | engine |
| f | fuel |
| G | governor |
| GB | gear box |
| i | insulation |
| j | jet |
| m | mean |
| N | nozzle |
| P | pitch line |
| T | turbine |
| t | tangential or throat |
| V | velocity |
| W | wheel |

## C. Straight Bleed System

A schematic diagram and a temperature-entropy diagram of the straight bleed system are shown in Figs. II-1 and II-2.

### 1. Derivation of the Head Loss Parameter, $\beta$, and the Dimensionless Pressure Drop Parameter, $X^*$

This derivation is concerned only with the duct and the flow in the duct; therefore, it may be used for both the bleed and burn and the straight bleed systems.

In designing a pneumatic transmission system, it is important to know the pressure drop in the duct. This makes it possible to compute the approximate energy available to the turbine at the end of the duct and to determine weight of air necessary to supply a given horsepower requirement.

The pressure drop in a duct is a function of the length and diameter of the duct, the number of fittings, the compressor bleed temperature and pressure and the weight of air flowing through the duct. Introduction of $\beta$ and $X^*$ in the expression for pressure drop simplifies the equation and provides a means for correlating the major duct variables with the duct inlet conditions.

The pressure drop in an air duct is due to frictional losses in the

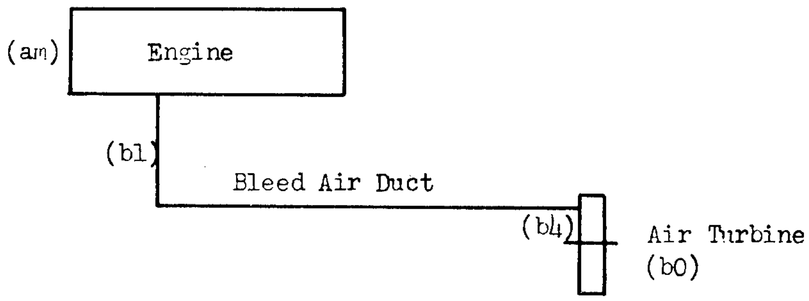


Fig. II-1 SCHEMATIC DIAGRAM FOR A BLEED AIR SYSTEM

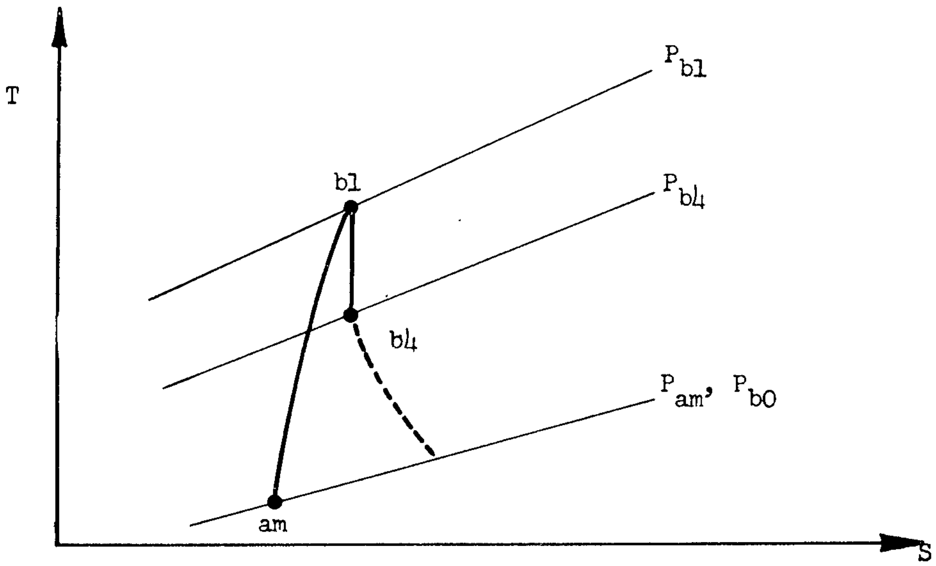


Fig. II-2 AIR CYCLE FOR BLEED AIR SYSTEM

various duct components such as straight sections, bends, elbows, valves, etc. The pressure drop in these duct components can be expressed in terms of pressure loss coefficients, $K_n$, defined as

$$K_n = \frac{\Delta P_n}{\frac{\gamma}{2g} V^2} \qquad \text{(II-1)}$$

where:

$K_n$ = head loss coefficient of any particular duct component

$\gamma$ = weight density of the air, $lb/ft^3$

$V$ = velocity of the air inside the duct, ft/sec

$\Delta P_n$ = pressure drop, $lb/ft^2$

Values of the coefficient $K_n$ are determined experimentally and are found in the literature, (Refs. II-1 and II-2).

For the straight portions of the duct the pressure loss is given by:

$$\Delta P = f \frac{L}{D} \frac{1}{2} \frac{\gamma}{g} V^2 = K_s \frac{\gamma}{2g} V^2 \qquad \text{(II-2)}$$

where:

$f$ = friction factor

$L$ = length of straight portion of duct, ft

$D$ = diameter of duct, ft

$K_s$ = pressure loss coefficient for straight pipe section

The value of the friction factor can be determined from charts such as the Moody Diagram, (Ref. II-3).

For the straight section of the duct, the K is a function of the diameter. However, since the influence of this quantity upon the $\Sigma K$ is small, its functional dependency on D can be neglected for this analysis.

The pressure loss due to an individual duct component is then given by:

$$\Delta P_n = K_n \frac{\gamma_n}{2g} V_n^2 \qquad \text{(II-3)}$$

The density of the air is given by the gas law as

$$\gamma_n = \frac{P_n}{R\ T_n} \tag{II-4}$$

where:

$R$ = gas constant, ft-lb/lb °R

$T$ = absolute temperature, °R

The velocity can be expressed in terms of weight rate of air flow, air density and the duct cross-sectional area ($A_n = D_n^2\ \pi/4$).

$$V_n = \frac{W_{BL}\ R\ T_n}{\pi/4\ D_n^2\ P_n} \tag{II-5}$$

Introducing Eqs. (II-4) and (II-5) into Eq. (II-3) yields:

$$\Delta P_n = K_n\ \frac{16R\ W_{BL}^2}{2g\pi^2}\ \frac{T_n}{P_n\ D_n^4} \tag{II-6}$$

The total pressure drop in a given duct system is the sum of the individual pressure drops of the component,

$$\Delta P = \frac{16R\ W_{BL}^2}{2g\pi^2} \sum \frac{K_n\ T_n}{P_n\ D_n^4} \tag{II-7}$$

The pressure, $P_n$, lies between $P_{bl}$ and $P_{bl}-\Delta P$, the exact value can only be determined by actual test of a given installation. For purposes of this analysis it will be assumed that $P_n$ is the mean pressure existing in the duct, that is

$$P_n = P_{bl} - \frac{1}{2}\ \Delta P \tag{II-8}$$

and the temperature is assumed constant and equal to the duct inlet temperature. This results in a slightly higher velocity and hence a somewhat larger pressure drop through the duct. Therefore, any error incorporated due to the approximation should introduce a safety factor. With this assumption Eq. (II-7) can be written as:

$$\Delta P = \frac{16R\, T_{b1}\, W_{BL}^{2}}{2g\pi^2 (P_{b1} - \frac{\Delta P}{2})\, D_1^4} \sum \left(\frac{D_1}{D_n}\right)^4 K_n \tag{II-9}$$

If the duct diameter is uniform, as is ordinarily the case, then Eq. (II-9) becomes:

$$\Delta P = \frac{16R\, T_{b1}\, W_{BL}^{2}}{2g\pi^2 (P_{b1} - \frac{\Delta P}{2})\, D^4} \Sigma K \tag{II-10}$$

Solving this equation for $\Delta P$ one obtains:

$$\Delta P = P_{b1} \left[ 1 - \sqrt{1 - \frac{16R\, T_{b1} \sum K}{g\pi^2\, P_{b1}^{2}} \frac{W_{BL}^{2}}{D^4}} \right] \tag{II-11}$$

The head loss parameter, $\beta$, is defined as:

$$\beta = \frac{16R\, T_{b1} \sum K}{g\pi^2 \quad P_{b1}^{2}} \tag{II-12}$$

The dimensionless pressure drop parameter, $X^*$, is defined as:

$$X^* = \sqrt{\beta}\, \frac{W_{BL}}{D^2} \tag{II-13}$$

Equation (II-11) can now be written as:

$$\frac{\Delta P}{P_{b1}} = 1 - \sqrt{1 - X^{*2}} \tag{II-14}$$

or since $\Delta P = P_{b1} - P_{b4}$,

$$\frac{P_{b4}}{P_{b1}} = \sqrt{1 - X^{*2}} \tag{II-15}$$

and

$$X^* = \sqrt{1 - \frac{P_{b4}}{P_{b1}}} \qquad \text{(II-16)}$$

2. <u>Derivation of the Dimensionless Horsepower Parameter, $HP^*$</u>

The dimensionless horsepower parameter, $HP^*$, is defined from the equation for turbine output horsepower. It is a convenient instrument for relating the required turbine horsepower with the duct characteristics, (duct diameter, flow rate, and pressure drop) and design conditions, (bleed air temperature and pressure, and ambient pressure).

The power output of a turbine expressed in terms of the turbine inlet conditions is given by:

$$HP = \frac{W_{BL}\ c_p\ \eta_T\ T_{b4}}{0.707} \left[ 1 - \left( \frac{P_{b0}}{P_{b4}} \right)^{\frac{k-1}{k}} \right] \qquad \text{(II-17)}$$

where:

$\frac{P_{b0}}{P_{b4}}$ = ratio of turbine exhaust pressure to turbine inlet pressure

$T_{b4}$ = turbine inlet temperature, °R

$W_{BL}$ = bleed air flow, lb/sec

0.707 = conversion factor, Btu/HP-sec

$\eta_T$ = turbine efficiency

The turbine inlet conditions, $T_{b4}$ and $P_{b4}$, can be expressed in terms of compressor outlet conditions, $T_{b1}$ and $P_{b1}$, and the duct losses as expressed by $\beta$ and D.

The temperature, $T_{b4}$, can be computed from

$$\frac{T_{b4}}{T_{b1}} = \left[ \frac{P_{b4}}{P_{b1}} \right]^{\frac{k-1}{k}} \qquad \text{(II-18)}$$

Substituting from Eq. (II-15) gives:

$$T_{b4} = T_{b1} \left(1 - X^{*2}\right)^{\frac{k-1}{2k}} \qquad \text{(II-19)}$$

The inverse pressure ratio, $\alpha$, is defined as:

$$\alpha = \frac{P_{b0}}{P_{b1}} \qquad \text{(II-20)}$$

Substituting Eqs. (II-15), (II-20), and (II-19) into Eq. (II-17) and assuming the specific heat of air is 0.24 Btu/lb °F, gives:

$$HP = \frac{W_{BL}\,\eta_T\,T_{b1}\,(1 - X^{*2})^{\frac{k-1}{2k}}}{2.95}\left[1 - \left(\frac{\alpha}{\sqrt{1 - X^{*2}}}\right)^{\frac{k-1}{k}}\right] \qquad \text{(II-21)}$$

From Eq. (II-13) the bleed air requirement, $W_{BL}$, can be expressed as:

$$W_{BL} = \frac{D^2}{\sqrt{\beta}}\,X^* \qquad \text{(II-22)}$$

This value of $W_{BL}$ can be introduced into Eq. (II-21) and by regrouping the terms the following dimensionless equation can be obtained:

$$\frac{2.95\,HP\,\sqrt{\beta}}{\eta_T\,T_{b1}\,D^2} = X^*\left[(1 - X^{*2})^{(k-1)/2k} - \alpha^{(k-1)/k}\right] \qquad \text{(II-23)}$$

The left hand side of Eq. (II-23) is defined as the dimensionless horsepower, $HP^*$, which can be rewritten using Eq. (II-12)

$$HP^* = \frac{2.95\sqrt{\beta}}{\eta_T\,T_{b1}\,D^2}\,HP = 4.84\,\frac{\sqrt{\Sigma K}}{\eta_T\,P_{b1}\sqrt{T_{b1}}\,D^2}\,HP \qquad \text{(II-24)}$$

Eq. (II-23) can now be written as:

$$HP^* = X^*\left[(1 - X^{*2})^{(k-1)/2k} - \alpha^{(k-1)/k}\right] \qquad \text{(II-25)}$$

The variation of $HP^*$ with $X^*$ for several values of $\alpha$ is shown as Fig. II-3.

### 3. Power Loss Due to Pressure Drop in the Duct

The effect of the energy loss due to the pressure drop in the duct

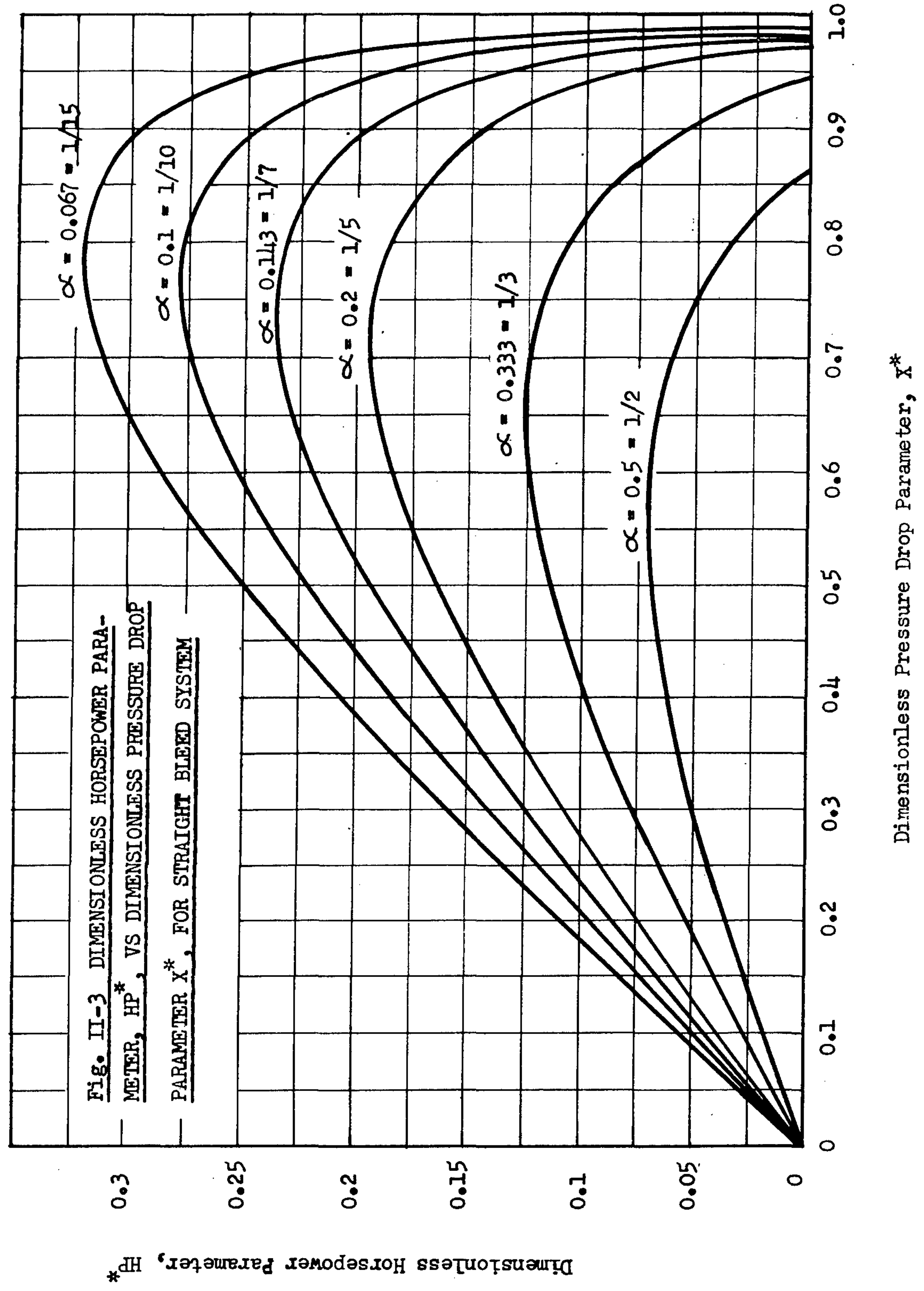


Fig. II-3 DIMENSIONLESS HORSEPOWER PARAMETER, HP*, VS DIMENSIONLESS PRESSURE DROP PARAMETER X*, FOR STRAIGHT BLEED SYSTEM

can be evaluated from Fig. II-3 in terms of power or bleed air flow. In the absence of pressure drops in the duct, $K = 1$, the turbine inlet conditions can be assumed to equal the compressor outlet conditions and the power relationship can be determined from Eq. (II-25) which reduces to the following approximation for small values of $X^*$ ($X^{*2}$ negligible as compared with unity):

$$HP^* = \left[1 - \alpha^{(k-1)/k}\right] X^* \tag{II-26}$$

This is the equation of the tangent to the $HP^*$ vs $X^*$ curve at $X^* = 0$.

The effect of duct losses on power output and air consumption is shown schematically in Fig. II-4. If the operating point of the system corresponds to point (1), then the power loss is proportional to the distance 1-3, where point (3) corresponds to the power output that could be obtained from the turbine maintaining the same air flow in the absence of duct losses.

The increase in bleed air requirement due to duct losses is proportional to the distance 1-2, where point (2) corresponds to the bleed air requirement of the turbine delivering the same power in the absence of duct losses.

### 4. Determination of Duct Diameter from the Dimensionless Horsepower Parameter

For a required power output, HP, a given duct configuration, $\Sigma K$, and the power available in the bleed air (expressed in $T_{b1}$ and $P_{b1}$), the duct diameter can be determined. Using the derivation of part 2 above, the required duct diameter can be obtained from Eq. (II-24), if the dimensionless horsepower parameter, $HP^*$, is known. $HP^*$ can be taken from Fig. II-3 if maximum permissible pressure drop is specified. From $\Delta P = P_{b4} - P_{b1}$, $X^*$ can b e calculated using Eq. (II-16) and the corresponding $HP^*$ value determined for a given value of $\alpha$. The duct diameter corresponding to a given $\Delta P$ then is:

$$D = \sqrt{\frac{2.95\sqrt{\beta}\ HP}{\eta_T\ T_{b1}\ HP^*}} \tag{II-27}$$

If the permissible pressure drop is not specified, the maximum value of $HP^*$, for the given $\alpha$, is taken from Fig. II-3. In this instance the duct diameter calculated from Eq. (II-27) is the smallest diameter capable of supplying enough air to satisfy the horsepower requirement.

### 5. Determination of the Bleed Air Flow from the Dimensionless Horsepower Parameter

The bleed air flow is important in computing the accessory fuel consumption and pressure drop. For a fixed system with a given $\Sigma K$ and engine

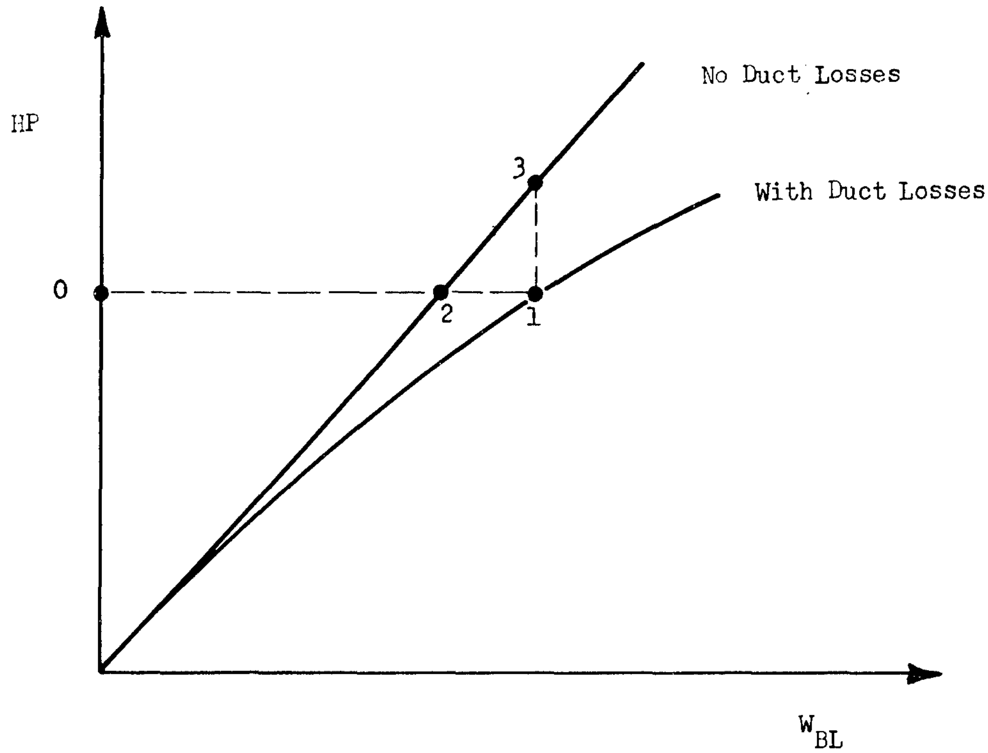


Fig. II-4

<u>POWER LOSS. AND INCREASE IN BLEED AIR REQUIREMENT DUE TO</u>

<u>DUCT LOSSES</u>

performance, the bleed air flow, $W_{BL}$, required for the power output, HP, and a duct diameter corresponding to a given duct pressure drop $\Delta P$ (or $X^*$) can be determined, from Eq. (II-22). Substituting the value of $\beta$ from Eq. (II-12) in Eq. (II-22) gives:

$$W_{BL} = \frac{\pi \sqrt{g}\; P_{bl}\; D^2}{4\sqrt{R\; T_{bl}\; \Sigma K}} \; X^* \tag{II-28}$$

or

$$W_{BL} = 0.61 \; \frac{P_{bl}\quad D^2}{\sqrt{T_{bl}\; \Sigma K}} \; X^* \tag{II-29}$$

The value of $X^*$ can be taken from Fig. II-3.

### 6. Derivation of the Specific Fuel Consumption for the Accessory System, $C''_{bl}$

Included in the total weight of an accessory system is the weight of fuel required to operate the accessories throughout the flight of the airplane. The specific fuel consumption indicates the amount of additional fuel a given engine must burn in order to supply the accessories with one pound of bleed air each second for one hour and provide required thrust.

This derivation is valid for both the straight bleed and the bleed and burn systems.

The specific fuel consumption of the accessory system is defined by:

$$C''_{bl} = \frac{W_F}{W_{BL}\,\tau} \tag{II-30}$$

where:

$W_F$ = weight of fuel required to operate accessory system for time, $\tau$, lb

$C''_{bl}$ = specific fuel consumption, lb of fuel per hr/lb of air per sec

$W_{BL}$ = bleed air, lb/sec

$\tau$ = duration of power extraction, hr

It is desired to develop an expression for $C''_{bl}$ in terms of engine operating conditions which can be calculated from available data.

The effect of bleed air extraction on the fuel consumption and thrust of the jet engine is shown schematically in Fig. II-5. When air is bled from the engine, the mass flow to the turbine is decreased, momentarily reducing the power available to the turbine. This tends to reduce its speed. However, since the engine control is primarily speed sensing, it acts to return the engine rpm to its initial value by increasing the fuel flow (point A to point B). The loss in mass flow also causes a decrease in thrust output of the engine. The power loss may be recovered by increasing the setting of the power control lever in the aircraft. This raises the base speed of the engine control and results in a further increase in fuel consumption (point B to point C).

Although not in accordance with Mil-E-5008, the following notation will be used in this report:

The changes in thrust and fuel consumption along a constant rpm line due to air bleed are:

$$\Delta F_n = C_{bl} \left( \frac{W_{BL}}{W_a} \right) F_n \qquad \text{(II-31)}$$

and

$$\Delta W_f = C_{bl}' \left( \frac{W_{BL}}{W_a} \right) W_f \qquad \text{(II-32)}$$

where

$C_{bl}'$, $C_{bl}$ = air bleed correction factors for fuel consumption and thrust respectively

$\Delta F_n$ = change in engine thrust, lb

$W_{BL}$ = weight of bleed air, lb/sec

$W_a$ = air flow through engine, lb/sec

$\Delta W_f$ = change in fuel consumption, lb/hr

$W_f$ = fuel consumption, lb/hr

From the engine specifications the change of net thrust is known for a given change of fuel flow. Fig. II-6 shows a plot of fuel consumption versus net thrust for a constant flight speed and an unburdened engine (no bleed air being extracted). In the normal operating range of the engine this plot is nearly a straight line. The slope of the line is given by:

$$\psi = \frac{d\,W_f}{d\,F_n} \qquad \text{(II-33)}$$

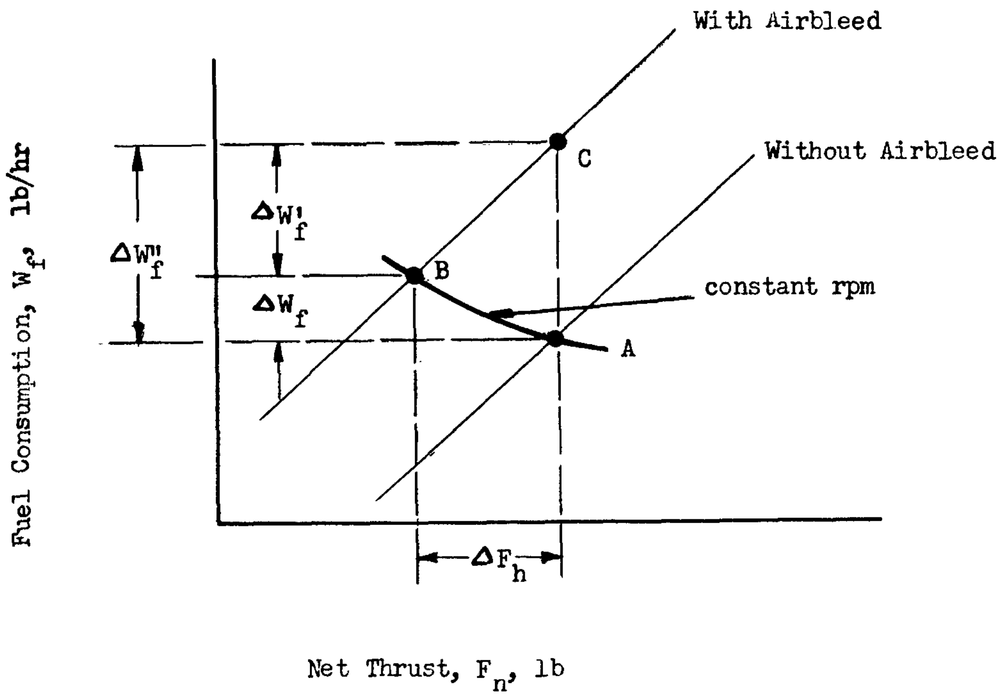


Fig. II-5 ENGINE OPERATING CONDITIONS

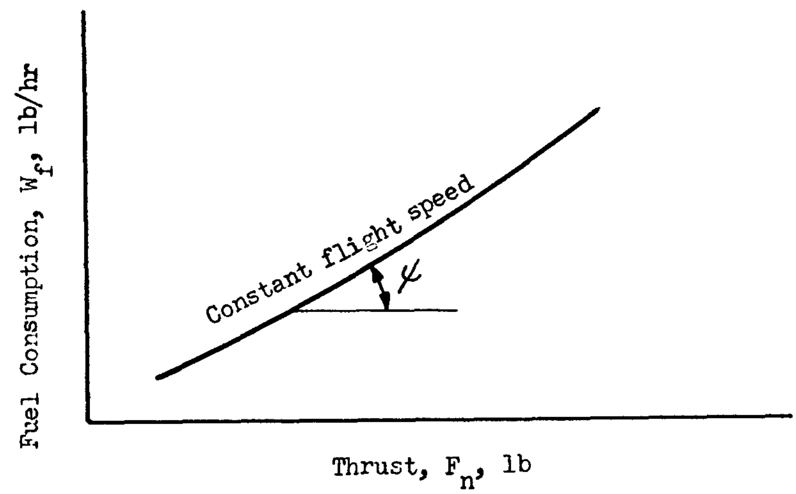


Fig. II-6 VARIATION OF FUEL FLOW WITH CHANGE OF THRUST FOR CONSTANT AIRPLANE SPEED

The change in fuel flow for a given change in thrust along a constant flight speed line is given by:

$$\Delta W'_f = \psi \Delta F_n \tag{II-34}$$

where:

$\Delta W'_f$ = change in fuel consumption, lb/hr

The net increase in fuel consumption due to bleeding of air from the compressor while maintaining a constant thrust, as shown in Fig. II-5, is given by

$$\Delta W_f'' = \Delta W_f + \Delta W_f' \tag{II-35}$$

where

$\Delta W_f''$ = total change in fuel flow due to air bleed, $\frac{\text{lb}}{\text{hr}}$

With consideration of Eqs. (II-31), (II-32), (II-34), this can be written as:

$$\Delta W''_f = \left( \frac{\psi C_{bl} \; F_n}{W_a} + \frac{C'_{bl} \; W_f}{W_a} \right) W_{BL} \tag{II-36}$$

Let

$$C''_{bl} = \frac{\psi \; C_{bl} \; F_n + C'_{bl} \; W_f}{W_a} \tag{II-37}$$

then

$$\Delta W''_f = C''_{bl} \; W_{BL} \tag{II-38}$$

The coefficient $C''_{bl}$ as defined by Eq. (II-37) is called the specific fuel consumption chargeable to the accessories and is determined from engine specifications, for any given operating condition of the engine.

7. Determination of the Optimum Duct Diameter

The total weight of the accessory system is written as:

$$\Sigma W = W_T + W_D + W_F + W_k \tag{II-39}$$

where:

$W_T$ = turbine weight, lb

$W_D$ = total duct weight including fittings and insulation

$W_F$ = total fuel weight, lb

$W_k$ = increment of total aircraft weight due to additional structural requirements and to the fuel required to overcome any increased aerodynamic drag chargeable to the transmission system, lb

The duct diameter for which the total weight, $\Sigma W$, becomes a minimum is called "optimum duct diameter".

For the derivation of the optimum duct diameter, it is assumed that the system is designed for the power requirements at cruise conditions and no overload capacity is necessary.

The usual method for minimizing such an equation would be to express the terms of the right side of Eq. (II-39) in terms of the duct diameter, differentiate, with respect to the diameter, and equate the resulting derivative to zero. The duct diameter which satisfies this condition is then the optimum duct diameter.

In this analysis, however, the dimensionless pressure drop parameter, $X^*$, is used as the independent variable instead of the duct diameter. When the optimum value of $X^*$ is determined, the optimum duct diameter can be obtained from $HP^*$ relationships.

It is first necessary to express the factors in Eq. (II-39) in terms of the independent variable, $X^*$. For this derivation, the turbine weight, $W_T$, and the increased structural weight and fuel weight, $W_k$, are assumed constant.

a. <u>Express the Duct Weight, $W_D$, in Terms of $X^*$</u>

The duct weight is given by:

$$W_D = \omega D L \qquad \text{(II-40)}$$

Since the duct weight coefficient, $\omega$, and the duct length, L, are constants, it remains to express the duct diameter in terms of $X^*$. Solving Eq. (II-23) for $D^2$ gives:

$$D^2 = \frac{2.95 \; HP \sqrt{\beta}}{\eta_T \; T_{bl}} \; \frac{1}{X^* \left[ (1 - X^{*2})^{(k-1)/2k} - \alpha^{(k-1)/k} \right]} \qquad \text{(II-41)}$$

By the following definitions:

$$\Phi = \frac{2.95\ HP}{\eta_T\ T_{bl}} \qquad \text{(II-42)}$$

and

$$\varphi(X^*, \alpha) = (1 - X^{*2})^{(k - 1)/2k} - \alpha^{(k - 1)/k} \qquad \text{(II-43)}$$

Equation (II-41) can be written as:

$$D = \frac{\sqrt[4]{\beta}\ \sqrt{\Phi}}{\sqrt{\varphi(X^*, \alpha)}\sqrt{X^*}} \qquad \text{(II-44)}$$

b. <u>Express the Total Weight of the Fuel, $W_F$, in Terms of $X^*$</u>

The weight of the fuel is given by:

$$W_F = C''_{bl}\ W_{BL}\ \tau \qquad \text{(II-45)}$$

where:

$C''_{bl}$ = specific fuel consumption of accessories, $\frac{\text{lb of fuel/hr}}{\text{lb of air/sec}}$

$W_{BL}$ = bleed air flow at cruise conditions, lb/sec

$\tau$ = duration of power extraction, hr

All of the terms in Eq. (II-45) are constants except the bleed air flow. To express the bleed air flow in terms of $X^*$, solve Eq. (II-44) for $D^2\ X^*/\sqrt{\beta}$. This gives:

$$\frac{D^2}{\sqrt{\beta}} X^* = \frac{\Phi}{\varphi(X^*, \alpha)} \qquad \text{(II-46)}$$

and from Eqs. (II-22), (II-42) and (II-43):

$$W_{BL} = \frac{\Phi}{\varphi(X^*, \alpha)} \qquad \text{(II-47)}$$

c. Differentiate the Total Weight Equation

The total weight equation may now be written as:

$$\Sigma W = W_T + W_k + \omega_L \frac{\sqrt[4]{\beta}\,\sqrt{\Phi}}{\sqrt{X^* \varphi(X^*, \alpha)}} + C''_{bl}\,\tau\,\Phi \frac{1}{\varphi(X^*, \alpha)} \tag{II-48}$$

In order to find the optimum value of $X^*$, (which is the value of $X^*$ that will minimize the total weight), Eq. (II-48) is differentiated with respect to $X^*$. **The derivative** is then equated to zero:

$$\frac{d\Sigma W}{d\,X^*} = \frac{\partial \Sigma W}{\partial X^*} + \frac{\partial \Sigma W}{\partial \varphi(X^*, \alpha)} \frac{\partial \varphi(X^*, \alpha)}{\partial X^*} = 0 \tag{II-49}$$

For this derivation, let,

$$C''_{bl}\,\Phi\,\tau = A \tag{II-50}$$

**and**

$$\omega_L \sqrt{\Phi}\sqrt[4]{\beta} = B \tag{II-51}$$

then,

$$\frac{\partial \Sigma W}{\partial X^*} = -\frac{B}{2}\frac{1}{X^{*3/2}\left[\varphi(X^*, \alpha)\right]^{1/2}} \tag{II-52}$$

$$\frac{\partial \Sigma W}{\partial \varphi(X^*, \alpha)} = -\frac{B}{2}\frac{1}{X^{*1/2}\left[\varphi(X^*, \alpha)\right]^{3/2}} - \frac{A}{\left[\varphi(X^*, \alpha)\right]^2} \tag{II-53}$$

and

$$\frac{\partial \varphi(X^*, \alpha)}{\partial X^*} = -\frac{k-1}{k} X^* \frac{1}{(1 - X^{*2})^{(k+1)/2k}} \tag{II-54}$$

Introducing these values into Eq. (II-49) and solving for $\frac{A}{B}$ gives:

$$\frac{A}{B} = \frac{1}{2}\sqrt{\frac{\varphi(X^*, \alpha)}{X^*}}\left[\frac{k}{k - 1}\,\frac{\varphi(X^*, \alpha)}{{X^*}^2}\,(1 - {X^*}^2)^{(k + 1)/2k} - 1\right] \qquad \text{(II-55)}$$

Let the dimensionless time parameter, $\tau^*$, be defined as:

$$\tau^* = \frac{A}{B} = \frac{C''_{bl}\,\sqrt{\Phi}}{\omega_L\,\sqrt[4]{\beta}} \qquad \text{(II-56)}$$

then,

$$\tau^* = \frac{1}{2}\sqrt{\frac{\varphi(X^*, \alpha)}{X^*}}\left[\frac{k}{k - 1}\,\frac{\varphi(X^*, \alpha)}{{X^*}^2}\,(1 - {X^*}^2)^{\frac{k + 1}{2k}} - 1\right] \qquad \text{(II-57)}$$

Equation (II-57) is shown graphically in Fig. II-7 for several values of $X^*$.

d. Determine the Optimum Duct Diameter

The dimensionless time parameter, $\tau^*$, can be calculated from design data with Eq. (II-56). Then the value of $X^*$ which satisfies Eq. (II-57) can be obtained from Fig. II-7. This is the optimum value of the dimensionless pressure drop parameter, $(X^*)_{opt}$. With this value of $(X^*)_{opt}$, the optimum dimensionless horsepower parameter, $(HP^*)_{opt}$, can be obtained from Fig. II-3. The optimum duct diameter can then be calculated from Eq. (II-27).

## D. Bleed and Burn System

A schematic diagram and a temperature-entropy diagram of the bleed and burn system are shown in Figs. II-8 and II-9, respectively.

The bleed and burn system is the same as the straight bleed system except that a combustion chamber is inserted just ahead of the air turbine. This slightly alters the equations for the dimensionless horsepower parameter, and the duct diameter. Consequently, these factors are derived below for a bleed and burn system.

### 1. Derivation of Dimensionless Horsepower Parameter, $HP^*$

For maximum efficiency the turbine inlet temperature is usually maintained at the maximum permitted by manufacturer's specifications. Therefore,

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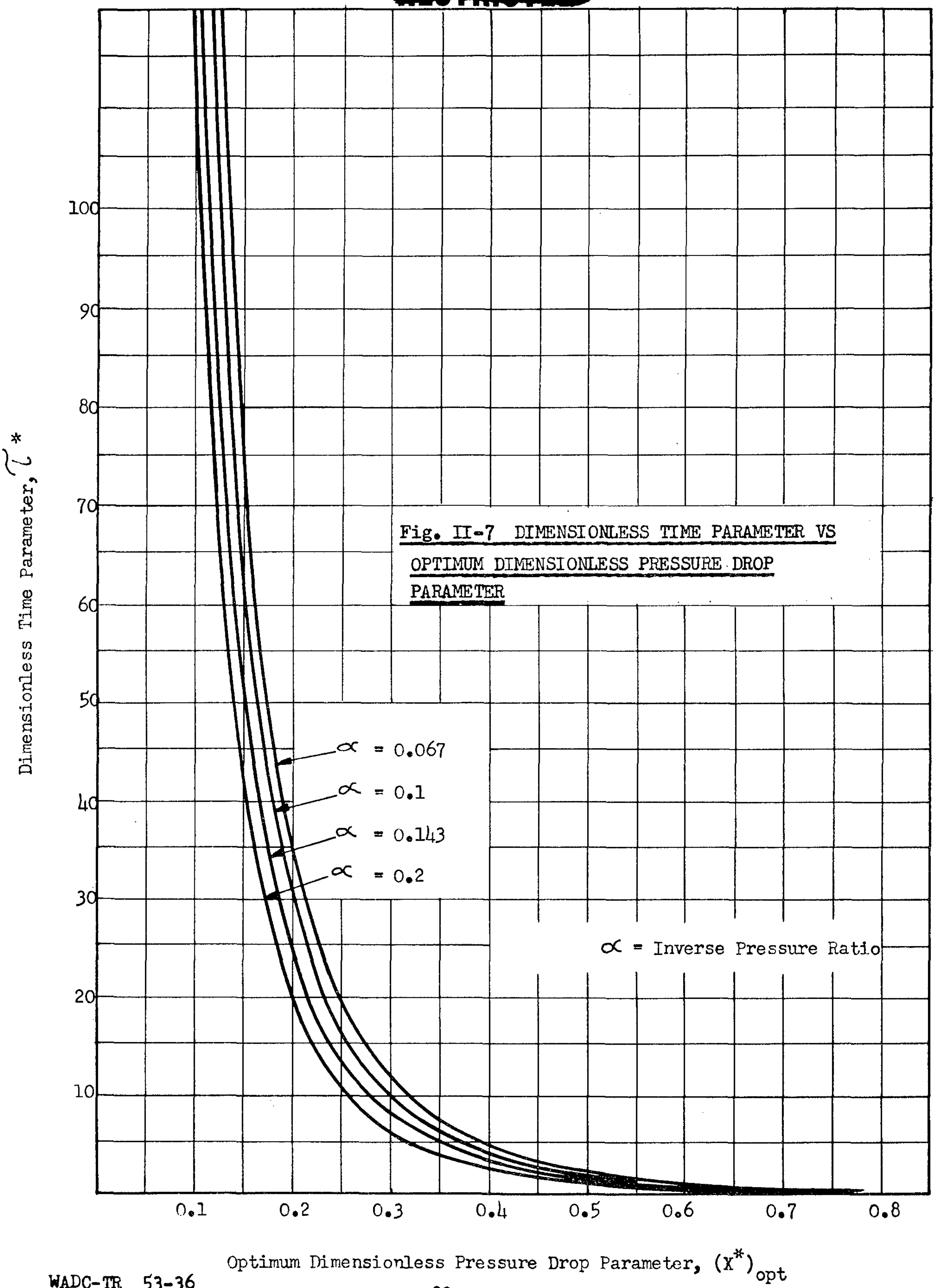


Fig. II-7 DIMENSIONLESS TIME PARAMETER VS OPTIMUM DIMENSIONLESS PRESSURE DROP PARAMETER



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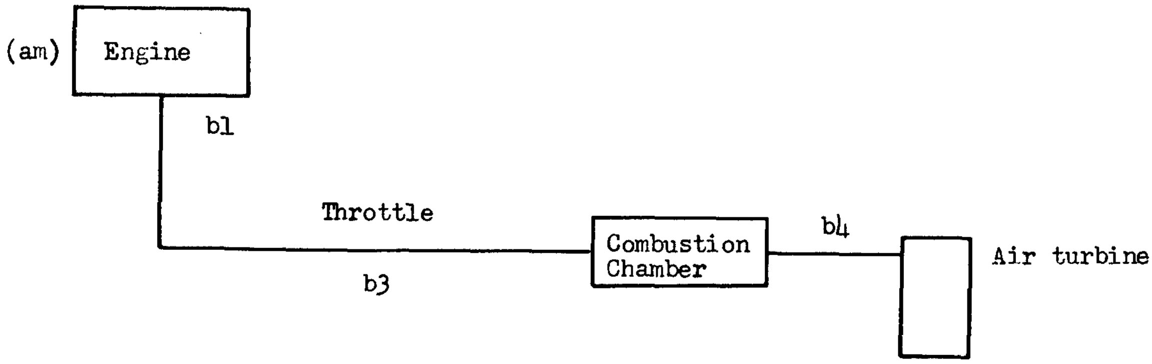


Fig. II-8 SCHEMATIC DIAGRAM OF A BLEED AND BURN SYSTEM

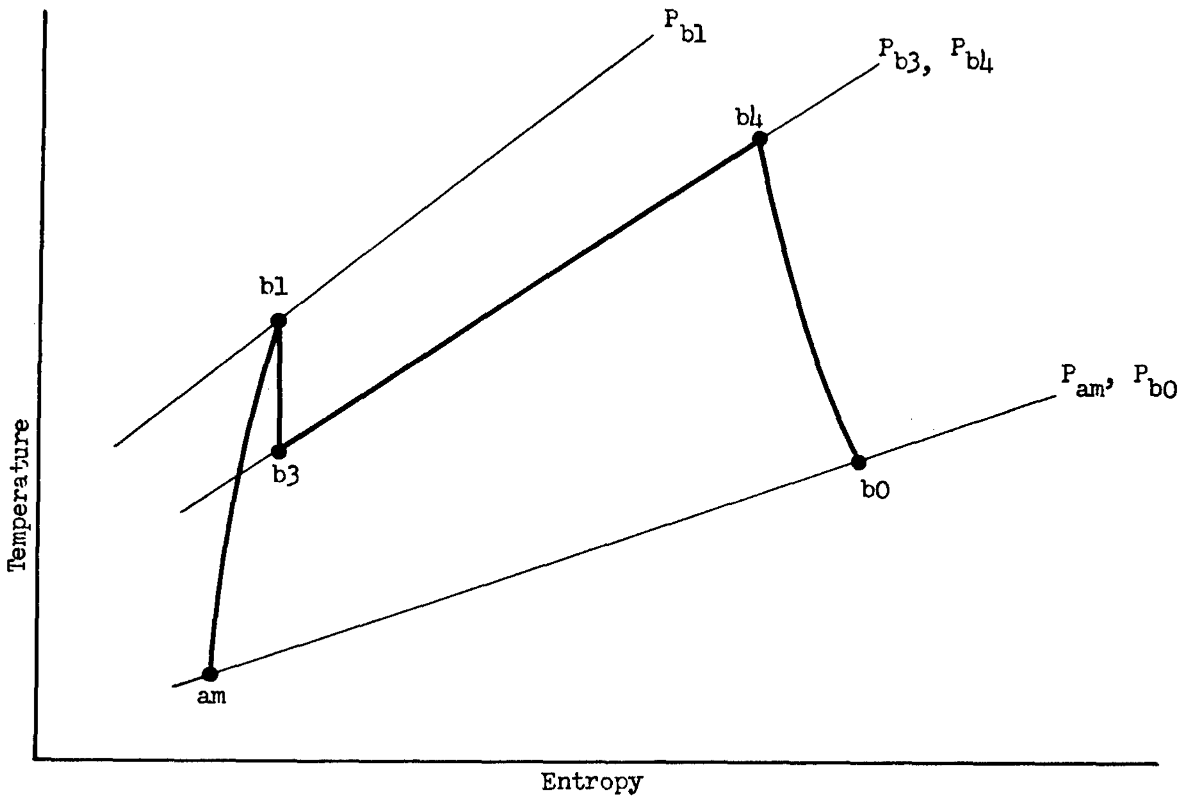


Fig. II-9 TEMPERATURE - ENTROPY DIAGRAM FOR BLEED AND BURN SYSTEM

the turbine inlet temperature, $T_{b4}$, is independent of the compressor discharge temperature. Substituting Eq. (II-15) into Eq. (II-17) the basic horsepower equation can be written as:

$$HP = \frac{W_{BL} \eta_T T_{b4}}{2.95} \left[ 1 - \left( \frac{\alpha}{\sqrt{1 - X^{*2}}} \right)^{\frac{k-1}{k}} \right] \qquad \text{(II-58)}$$

Substituting Eq. (II-22) for $W_{BL}$ gives:

$$\frac{2.95\sqrt{\beta}\, HP}{\eta_T D^2 T_{b4}} = X^* \left[ 1 - \left( \frac{\alpha}{\sqrt{1 - X^{*2}}} \right)^{\frac{k-1}{k}} \right] \qquad \text{(II-59)}$$

The left member of Eq. (II-59) is defined as the dimensionless horsepower parameter, $HP^*$.

$$HP^* = \frac{2.95\sqrt{\beta}}{D^2 \eta_T T_{b4}} HP \qquad \text{(II-60)}$$

Then,

$$HP^* = X^* \left[ 1 - \left( \frac{\alpha}{\sqrt{1 - X^{*2}}} \right)^{\frac{k-1}{k}} \right] \qquad \text{(II-61)}$$

Equation (II-61) is shown graphically in Fig. II-10 for several values of $X^*$.

2. <u>Determination of the Duct Diameter and Bleed Air Flow from Dimensionless Horsepower Parameter</u>

If the horsepower parameter, $HP^*$, is known, the duct diameter can be computed from Eq. (II-60) which can be written as:

$$D = \sqrt{\frac{2.95\sqrt{\beta}\, HP}{\eta_T T_{b4} HP^*}} \qquad \text{(II-62)}$$

The bleed air flow can then be calculated from Eq. (II-13). The diameter, D, shown in this equation, is determined from Eq. (II-62), and the value of $X^*$ can be taken from Fig. II-10.

## E. Weight of the Air Turbine

In evaluating a pneumatic system it is necessary to estimate the weight

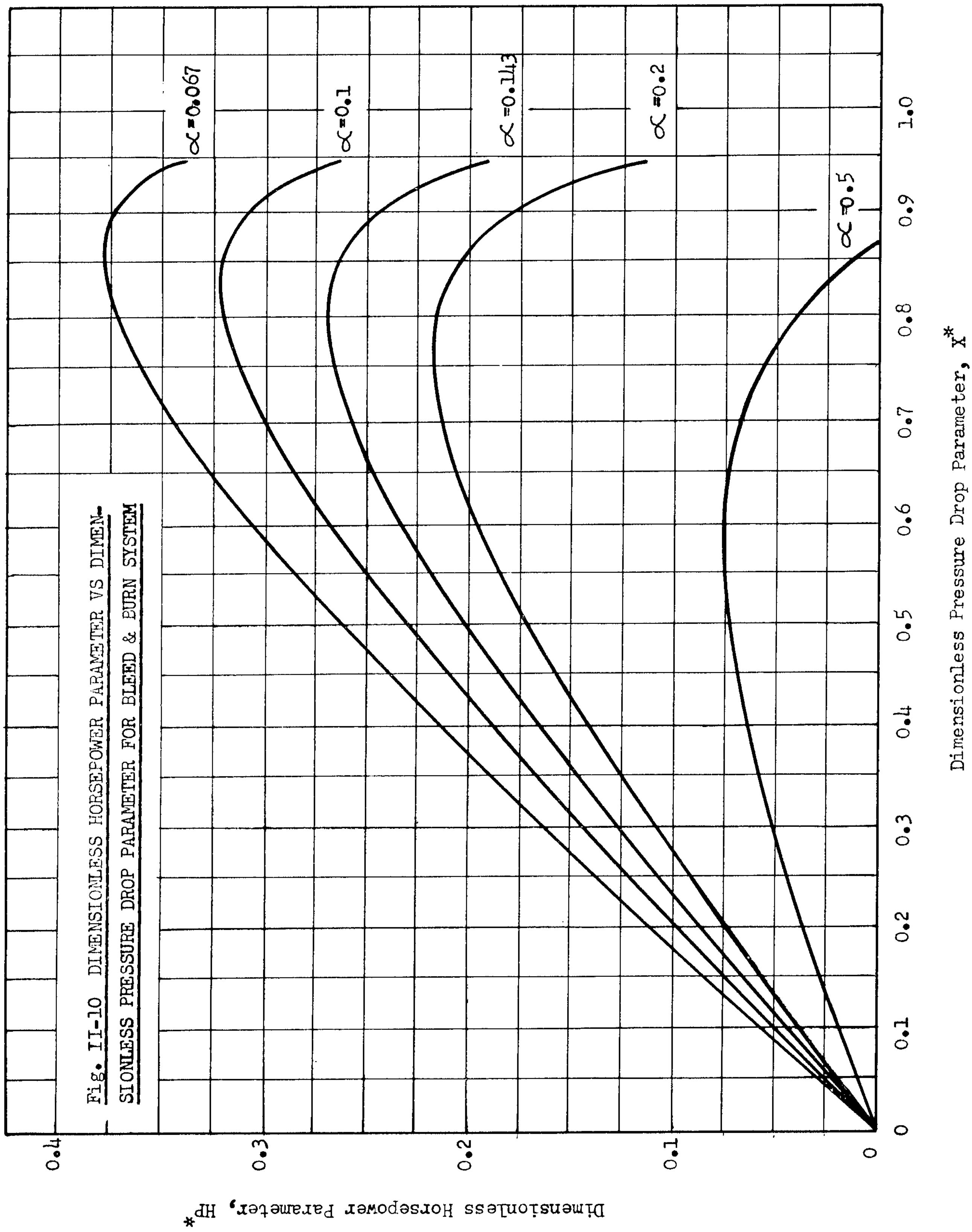


Fig. II-10 DIMENSIONLESS HORSEPOWER PARAMETER VS DIMENSIONLESS PRESSURE DROP PARAMETER FOR BLEED & BURN SYSTEM

of the turbine. The size and weight of a pneumatic turbine are based initially on the nozzle throat area and jet velocity of the gas leaving the nozzle.

The wheel diameter of an axial flow turbine is determined from the nozzle area and the jet velocity. The size and weight of the wheel and casing are in turn based on the wheel diameter.

Nozzle throat area and jet velocity equations are derived for a straight bleed system in sections 1 and 2 below. The nozzle throat area for a bleed and burn system is derived in section 3 below. Equations of casing weight, $W_C$, and the combined wheel and casing weight, $W_{WC}$, are derived for an axial flow turbine in sections 4 and 5 below.

1. <u>Derivation of Nozzle Throat Area, $A_t$, for a Straight Bleed System</u>

The following derivation makes it possible to express the nozzle throat area of a turbine in terms of the known duct characteristics, $\Sigma K$, D, and $X^*$.

The throat area required to pass a given amount of air is given by Ref. II-8.

$$A_t = \frac{W_{BL}}{0.532} \frac{\sqrt{T_{b4}}}{P_{b4}} \qquad \text{(II-63)}$$

The pressure and temperature can again be expressed in terms of the compressor outlet conditions. Using Eqs. (II-15) and (II-19) gives:

$$A_t = \frac{W_{BL}}{0.532} \frac{\sqrt{T_{b1}}}{P_{b1}} \frac{\left(1 - X^{*2}\right)^{(k-1)/4k}}{\left(1 - X^{*2}\right)^{1/2}}$$

or

$$A_t = \frac{W_{BL}\sqrt{T_{b1}}}{0.532\, P_{b1}} \frac{1}{\left(1 - X^{*2}\right)^{(k+1)/4k}} \qquad \text{(II-64)}$$

Introducing the value of $W_{BL}$ as given by Eq. (II-28),

$$A_t = \frac{\pi \sqrt{g}\, D^2}{4\sqrt{R}\sqrt{\Sigma K}\; 0.532} \frac{X^*}{\left(1 - X^{*2}\right)^{\frac{k+1}{4k}}} \qquad \text{(II-65)}$$

$$A_t = 1.148 \frac{D^2}{\sqrt{\Sigma K}} \frac{X^*}{\left(1 - X^{*2}\right)^{\frac{k+1}{4k}}} \qquad \text{(II-66)}$$

Defining the dimensionless pressure drop function for the straight bleed system, $g(X^*)$, as

$$g(X^*) = \frac{X^*}{(1 - X^{*2})^{\frac{k+1}{4k}}} \qquad \text{(II-67)}$$

Eq. (II-66) becomes:

$$A_t = 1.148 \frac{D^2}{\sqrt{\Sigma K}} g(X^*) \qquad \text{(II-68)}$$

The function $g(X^*)$ is shown as Fig. II-11.

Under some operating conditions, such as partial load at low altitude and high thrust output of the main engine, the values of $X^*$ become very small and the use of Figs. II-3 and II-8 becomes difficult. For this range the following approximations can be used:

For Eq. (II-25)

$$HP^* = (1 - \alpha^{\frac{k-1}{k}}) X^* \qquad \text{(II-69)}$$

and for Eq. (II-68)

$$A_t = 1.148 \frac{D^2}{\sqrt{\Sigma K}} X^* \qquad \text{(II-70)}$$

Eqs. (II-69) and (II-70) show that for small values of $X^*$ the dimensionless power $HP^*$ and the throat area are linear functions of the variable $X^*$.

## 2. Jet Velocity at the Exit of a Convergent Nozzle

The following derivation results in an expression for the jet velocity in terms of the bleed air temperature which is determined from the operating conditions of the engine.

For a perfect gas, the jet velocity at the exit of a nozzle is given by:

$$V_j = \sqrt{2gJ\, c_p\, \Delta T} \qquad \text{(II-71)}$$

where:

$J$ = mechanical equivalent of heat, $\frac{\text{Ft-lb}}{\text{Btu}}$

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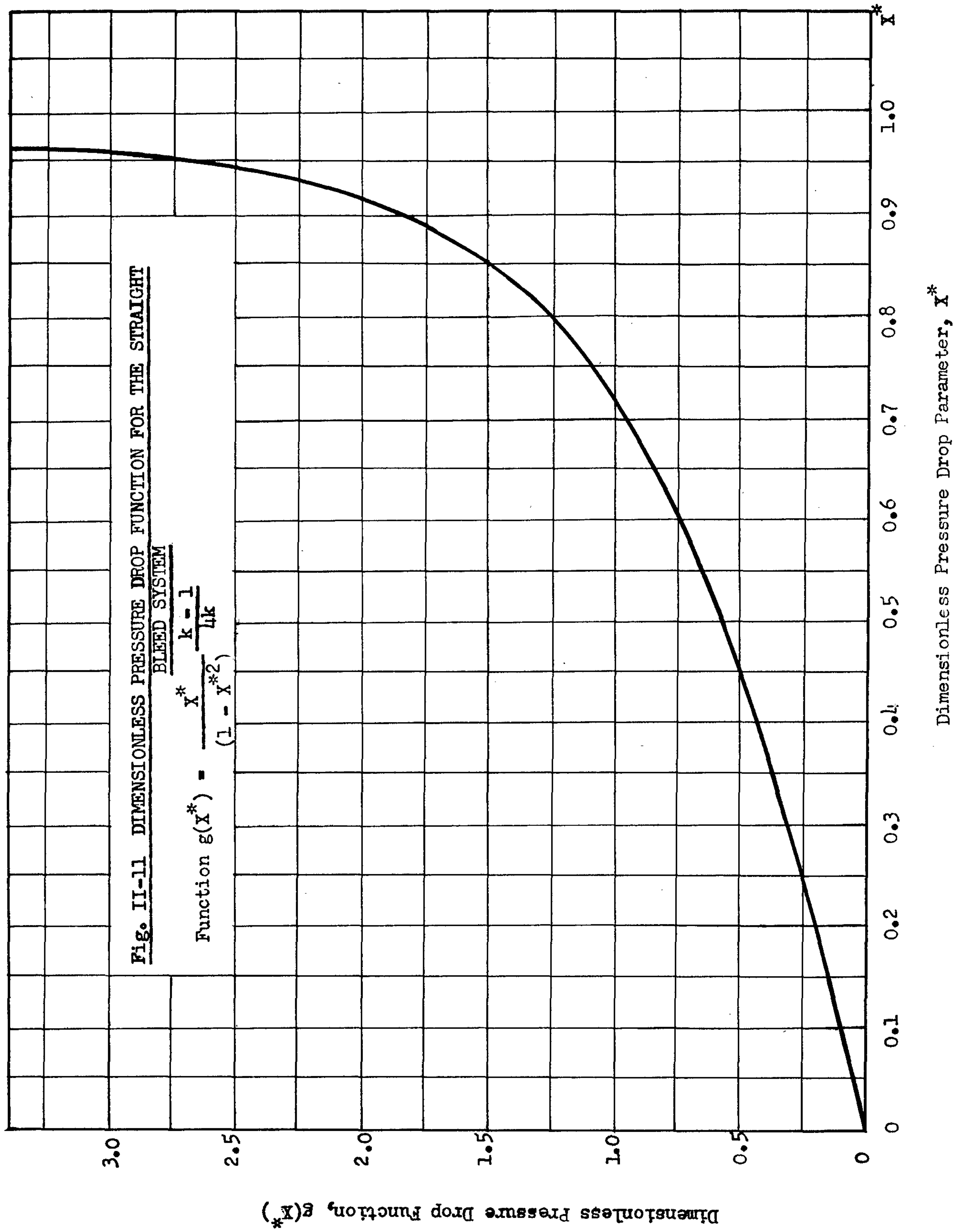


**Fig. II-11 DIMENSIONLESS PRESSURE DROP FUNCTION FOR THE STRAIGHT BLEED SYSTEM**

Function $g(X^*) = \dfrac{X^*}{(1 - X^{*2})^{\frac{k-1}{4k}}}$



[redacted]

$$c_p = \text{specific heat at constant pressure, } \frac{\text{Btu}}{\text{lb } ^\circ\text{F}}$$

$$\Delta T = \text{temperature drop across the nozzle, } ^\circ\text{R}$$

The temperature drop across the nozzle can be written as:

$$\Delta T = T_{b4}\,(1 - r_T) \tag{II-72}$$

where:

$r_T$ = ratio of temperature at nozzle exit to the entrance temperature

The temperature ratio, $r_T$, can be expressed in terms of the pressure ratio, r, by:

$$r_T = r^{(k-1)/k} \tag{II-73}$$

Equations (II-73) and (II-72) can now be introduced into Eq. (II-71), and since

$$c_p = (k/k - 1)(R/J), \tag{II-74}$$

$$V_j = \sqrt{\frac{k}{k-1}\, 2\, g\, RT_{b4}}\, \sqrt{1 - r^{(k-1)/k}} \tag{II-75}$$

When the pressure ratio across a converging nozzle is equal to the critical pressure ratio, then:

$$r = r_{cr} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \tag{II-76}$$

and

$$1 - r^{(k-1)/k} = \frac{k-1}{k+1} \tag{II-77}$$

With this expression, Eq. (II-74) reduces to

$$V_j = \sqrt{\frac{k}{k+1}\, 2g\, R\, T_{b4}} \quad \text{for} \left[ r = r_{cr} \right] \tag{II-78}$$

This velocity corresponds to sonic velocity at the nozzle exit. If the pressure ratio is larger than the critical ratio, the exit velocity is subsonic and is given by Eq. (II-75) which can be written as:

$$V_j = C_u \sqrt{\frac{k}{k+1} \, 2 \, g \, R \, T_{b4}} \qquad \text{(II-79)}$$

where:

$$C_u = \sqrt{\frac{k+1}{k-1}} \sqrt{1 - r^{(k-1)/k}} \quad \text{for} \quad \left[ r > r_{cr} \right] \qquad \text{(II-80)}$$

When the pressure ratio is less than the critical ratio, uncontrolled expansion of the gases occurs after exit of a converging nozzle. This uncontrolled expansion takes place at low efficiencies, and the extent of the free expansion of the gases depends on the geometry of the nozzle bank, (Ref. II-6).

The jet velocity for this case is given by Eq. (II-75), which can be written as:

$$V_j = C_u \sqrt{\frac{k}{k+1} \, 2g \, R \, T_{b4}} \qquad \text{(II-81)}$$

where:

$$C_u = C_x \sqrt{\frac{k+1}{k-1}} \sqrt{1 - r^{(k-1)/k}} \quad \text{for} \quad \left[ r < r_{cr} \right] \qquad \text{(II-82)}$$

$C_x$ = free expansion coefficient

The coefficient, $C_x$, can be determined experimentally for a given nozzle bank. The variation of the coefficient, $C_u$, with the pressure ratio, $r$, is shown schematically in Fig. II-12. For design conditions the ratio, $r$, will be less than critical, and the coefficient, $C_u$, can be taken equal to unity.

Since the turbine inlet temperature, $T_{b4}$, is independent of the compressor discharge temperature for a bleed and burn system, Eq. (II-81) may be used to determine the jet velocity for a bleed and burn system. The temperature $T_{b4}$ in Eq. (II-81) can be expressed in terms of the compressor inlet temperature for a straight bleed system. Using Eq. (II-19),

$$\sqrt{T_{b4}} = \sqrt{T_{b1}} \left(1 - X^{*2}\right)^{k - 1/4k} \qquad \text{(II-83)}$$

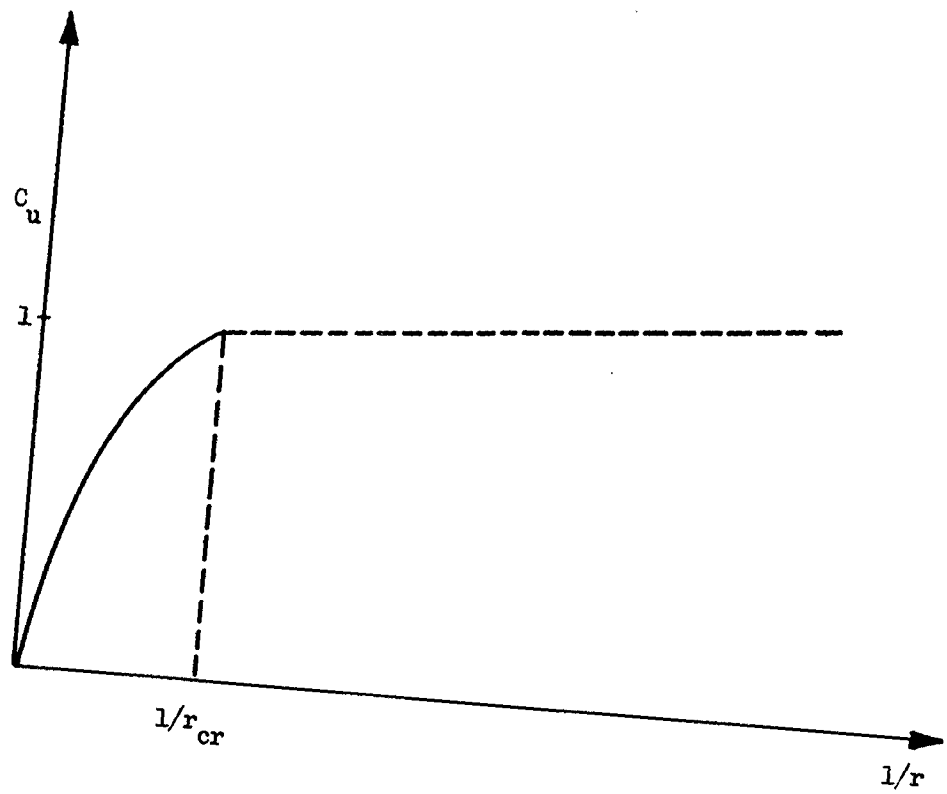


Fig. II-12 VARIATION OF THE VELOCITY COEFFICIENT $C_u$ WITH THE INVERSE OF THE PRESSURE RATIO

Since $X^*$ is smaller than one, $(1 - X^{*2})$ is less than one. The value of k can be taken as 1.4. With this value for k,

$$\frac{k - 1}{4k} = \frac{0.4}{5.6} = \frac{1}{14}$$

That is, the fourteenth root must be extracted **from a number smaller than one.** For the range of values of $X^*$ encountered this can be assumed to be equal to one.

Using these two approximations, that is, $C_u(1 - X^{*2})^{k - 1/4k} = 1$, Eqs. (II-78) and (II-81) can be written as:

$$V_j = \sqrt{\frac{k}{k + 1}\, 2g\, R\, T_{b1}} \quad \text{for} \quad \left[r \le r_{cr}\right] \tag{II-84}$$

The jet velocity at the exit of the nozzle can be evaluated from the compressor discharge data.

3. <u>Derivation of Nozzle Throat Area, $A_t$, for a Bleed and Burn System</u>

The air flow through the turbine nozzles is given by:

$$W_{BL} = \frac{0.532\, A_t\, P_{b4}}{\sqrt{T_{b4}}} \tag{II-85}$$

The air flow can also be expressed in terms of the dimensionless pressure drop parameter, $X^*$. See Eq. (II-13). Substitute Eq. (II-13) in Eq. (II-85).

$$\frac{D^2}{\sqrt{\beta}}\, X^* = \frac{0.532\, A_t\, P_{b4}}{\sqrt{T_{b4}}} \tag{II-86}$$

Substitute from Eq. (II-15) and solve for $A_t$. Then,

$$A_t = \frac{\sqrt{T_{b4}}\, D^2}{0.532\, P_{b1}\sqrt{\beta}} \; \frac{X^*}{\sqrt{1 - X^{*2}}} \tag{II-87}$$

The dimensionless pressure drop function for the bleed and burn system, $h(X^*)$,

is defined as:

$$h(X^*) = \frac{X^*}{\sqrt{1 - X^{*2}}} \qquad \text{(II-88)}$$

The function, $h(X^*)$ is shown graphically in Fig. II-13.

Substituting this definition in Eq. (II-87) gives:

$$A_t = \frac{\sqrt{T_{b4}}\; D^2}{0.532\; P_{b1}\sqrt{\beta}}\, h(X^*) \qquad \text{(II-89)}$$

or using the definition for $\beta$ given by Eq. (II-12),

$$A_t = 1.15\, \frac{D^2}{\sqrt{\Sigma K}} \sqrt{\frac{T_{b4}}{T_{b1}}}\, h(X^*) \qquad \text{(II-90)}$$

4. <u>Derivation of Turbine Casing Weight, $W_C$</u>

The weight of the turbine casing can be approximated by replacing the casing with a cylindrical enclosure having the same diameter as the tip of the turbine wheel.

$$W_C = \pi d\, w_C\, t\, \gamma + 2\, \frac{\pi}{4} d^2 \gamma t \qquad \text{(II-91)}$$

where:

$w_C$ = width of the casing, in.

d = tip diameter of the turbine wheel, in.

t = thickness of turbine casing, in.

$\gamma$ = weight density of casing material, lb/in$^3$

Eq. (II-91) can be written as:

$$W_C = \pi d^2 \gamma t \left( \frac{w_C}{d} + \frac{1}{2} \right) \qquad \text{(II-92)}$$

The wheel diameter can be expressed in terms of the annular area, A.

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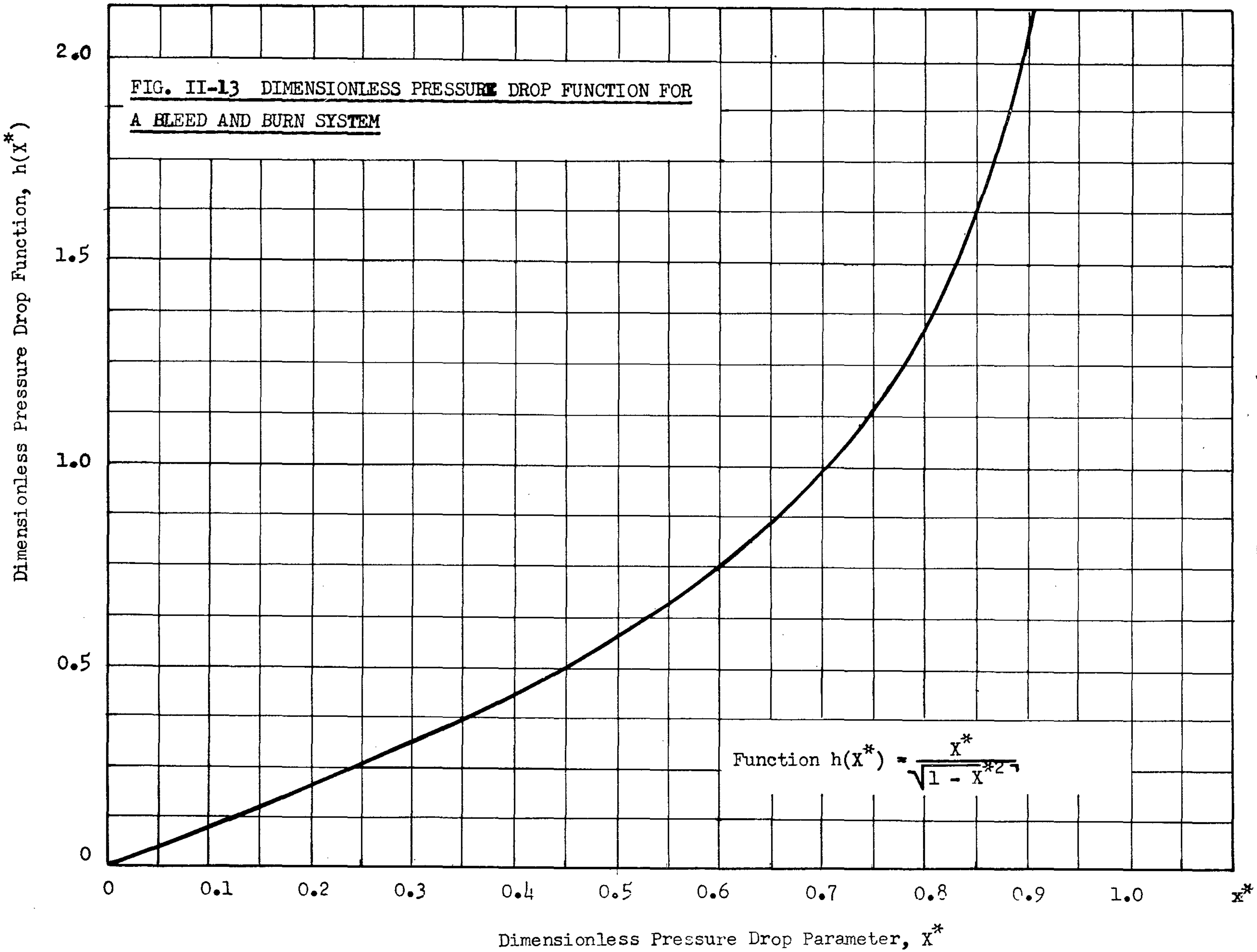
FIG. II-13 DIMENSIONLESS PRESSURE DROP FUNCTION FOR A BLEED AND BURN SYSTEM



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and the ratio of blade height to mean wheel radius, $h/r_m$, as follows:

$$d = 2r_m + h = 2r_m\,(1 + h/2r_m) \tag{II-93}$$

and since

$$A = 2\pi r_m h \tag{II-94}$$

then

$$2r_m = A/\pi h \tag{II-95}$$

Equation (II-92) can now be written as:

$$W_C = \pi\gamma t\left(\frac{w_C}{d} + \frac{1}{2}\right)\frac{2r_m A}{\pi h}\,(1 + h/2r_m)^2 \tag{II-96}$$

or

$$W_C = A\;t\gamma\left(\frac{w_C}{d} + \frac{1}{2}\right)\frac{(1 + h/2r_m)^2}{h/2r_m} \tag{II-97}$$

5. Derivation of Combined Weight of Turbine Wheel and Casing, $W_{WC}$

The turbine wheel weight is given by Eq. (II-39) of Part 1 as:

$$W_W = A\,h/\delta\;F(h/r_m) \tag{II-98}$$

where:

$W_W$ = wheel weight, lb

$\delta$ = aspect ratio, blade height/blade width

$h$ = blade height, in.

$F(h/r_m)$ = a function of blade height to mean radius ratio (see Ref. II-5)

The combined weight is given by the sum of Eqs. (II-98) and (II-97).

$$W_{WC} = W_W + W_C = \frac{hA\;\;F(h/r_m)}{\delta} + At\gamma\left(\frac{w_C}{d} + \frac{1}{2}\right)\frac{(1 + h/2r_m)^2}{2h/r_m} \tag{II-99}$$

The annular area can be expressed as follows:

$$A = 2\pi r_m h = 2\pi h^2 \, (r_m/h) \tag{II-100}$$

Solving for h gives:

$$h = \sqrt{\frac{A}{2\pi}} \sqrt{\left(\frac{h}{r_m}\right)} \tag{II-101}$$

Substituting in Eq. (II-99),

$$W_{WC} = \frac{F(h/r_m) \quad h/r_m}{\sqrt{2\pi}\;\delta} A^{3/2} + \frac{t\,\gamma\left(\frac{w_C}{d} + \frac{1}{2}\right)\left(1 + \frac{h}{2r_m}\right)^2}{h/2r_m} A \tag{II-102}$$

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HYDRAULIC POWER TRANSMISSION SYSTEMS

Section III

A. Introduction

The material presented in this section consists of derivations of equations used in the analysis of the hydraulic power transmission system. **In most cases where the result has been shown graphically in Part 1 of this** report, the respective figure is repeated here for easier reference.

B. Nomenclature

| | |
|---|---|
| $A$ | parameter representing the fixed weight of a constant flow variable pressure system, defined by Eq. (III-110), lb |
| $A_c$ | face area of the oil cooler, $in^2$ |
| $A_m$ | area of metal in transmission line cross-section, $ft^2$ |
| $a$ | flow area of transmission line cross-section, $ft^2$ |
| $B$ | parameter in the total weight equation of a constant flow, variable pressure system defining the increase in weight of the system due to system inefficiency, defined by Eq. (III-111) |
| $b$ | constant in determining the weight of a pump-motor combination |
| $C$ | parameter in total weight equation of a constant flow, variable pressure system defining the weight, the transmission lines and the reservoir, defined by Eq. (III-112) |
| $c$ | constant in determining the weight of a pump-motor combination |
| $C_{PX}$ | thrust correction factor of engine due to power extraction |
| $C'_{PX}$ | fuel flow correction factor of engine due to power extraction |
| $C''_{PX}$ | specific fuel consumption of the engine for the increment of total engine power which is extracted by the power transmission system, lbs/hr/HP |
| $D$ | inside diameter of hydraulic transmission lines, ft |
| $D_o$ | outside diameter of hydraulic transmission lines, ft |
| $D^*$ | non-dimensional parameter representing the inside diameter of the hydraulic transmission line |
| $F_n$ | jet engine thrust at design cruise conditions of the airplane, lbs |

$f$ friction factor from Moody diagram

$G$ energy parameter, $sec^2/ft^2$, defined by Eq. (III-7)

$g$ acceleration due to gravity, $ft/sec^2$

$H$ parameter representing the fixed weight of a constant pressure, variable flow system, and defined by Eq. (III-117)

$HP$ power output of the system

$HP_a$ power output of the pump

$HP_{cr}$ power requirements from system at design cruise conditions

$HP_m$ maximum power required from system including overloads

$HP_n$ power required by driven accessories at their normal rating

$HP_r$ rated power output of a pump-motor combination at rated pressure

$HP_{REF}$ a reference power used for comparison of jet engine performance at any operating condition and arbitrarily chosen as 10 per cent of the jet power at sea level static conditions

$HP_{EXT}$ total power extracted from engine by the power transmission system

$\Delta HP_L$ power losses due to line inefficiency

$\Delta HP_u$ power losses due to pump and motor inefficiency

$\Sigma \Delta HP$ total power losses of the system

$h$ ratio of the weight of the filled reservoir to the weight of the fluid contained by it

$J$ average weight per unit length of the auxiliary and control lines, lb/ft

$K$ velocity heads loss of the individual fitting unit, or bend indicated by its appropriate subscript

$\Sigma K$ total velocity heads loss in the transmission line system

$L$ total length of the hydraulic transmission lines, ft

$M$ parameter defined by Eq. (III-119) and representing the weight of fuel required by engine due to pump and motor inefficiency in a constant pressure, variable flow system

$m$ ratio of the weight of a complete hydraulic line, including fittings, to the weight of a bare tube of equivalent length

$N_a$ design rated speed of pump, rpm

$N_b$ design rated speed of motor, rpm

$N_r$ design rated speed of combination (same speed as the rated speed of the motor), rpm

$P$ pressure difference between the ports of the pump, $lb/ft^2$

$P_b$ pressure difference between the ports of the motor, $lb/ft^2$

$P_m$ maximum pressure difference between ports of the pump, $lb/ft^2$

$P_o$ operating pressure of pump-motor combination, used to determine the weight of the combination, $lb/ft^2$

$P_r$ rated operating pressure of pump-motor combination, $lb/ft^2$

$P_s$ maximum gage pressure of system - this is the maximum pump pressure plus any supercharge pressure, $lb/ft^2$

$\Delta P$ pressure loss due to transmission line inefficiency, $lb/ft^2$

$Q$ flow rate in system, $ft^3/sec$

$Q^*$ non-dimensional parameter representing the flow rate

$R$ parameter defined by Eq. (III-118), and representing the increment in weight of constant pressure, variable flow system due to increased pump, motor and oil cooler capacity which is required because of system inefficiency

$S$ parameter defined by Eq. (III-120), and representing the weights of the transmission lines and reservoir of a constant pressure, variable flow system

$s$ maximum permissible working stress of the metal in the transmission line wall, $lbs/ft^2$

$t$ thickness of transmission line wall, ft

$V$ fluid velocity in the hydraulic transmission lines, ft/sec

$V^*$ non-dimensional parameter representing the fluid velocity in the hydraulic transmission lines

$V_L$ volume of fluid contained in the lines, $ft^3$

$V_r$ volume of fluid contained in the reservoir, $ft^3$

$W$ weight, lb

$\Sigma W$ total weight of the hydraulic transmission system, lb

$W_a$ weight of pump, lb

$W_b$ weight of motor, lb

$W_c$ weight of oil cooler, lb

$W_{CL}$ weight of auxiliary and control lines, lb

$W_F$ weight of fuel, consumed by engine to deliver the power extracted by the accessory drive system, lb

$W_f$ fuel flow rate to engine at design cruise conditions, without accessory drive system operating, lb/hr

$W_{fr}$ weight of fluid in reservoir, lb

$W_k$ weight increment of airplane due to any structural requirements chargeable to the system as well as the fuel required to overcome any aerodynamic drag chargeable to the system, lb

$W_L$ weight of transmission lines and fluid, lb

$W_l$ weight of fluid in the transmission lines, lb

$W_m$ weight of tubing in transmission lines, lb

$W_r$ weight of reservoir and enclosed fluid, lb

$W_s$ weight of pump-motor combination which, when closely coupled, has the same power output as that desired from system, lb

$\Delta W_s$ weight of pump-motor increment required for overcoming line losses, lb

$W_u$ weight of pump-motor combination required by the system, lb

$x^*$ pressure loss coefficient evaluated at cruise power output conditions

$x^*_m$ pressure loss coefficient evaluated at maximum power output conditions

$z$ function of $x^*$ defined by Eq. (III-129)

$\alpha$ fraction of hydraulic transmission line fluid volume carried in the reservoir for fluid expansion and de-aeration purposes

$\beta$ fraction of transmission line fluid volume carried in the reservoir per hour of power extraction to compensate for small seepage leaks

$\gamma_f$ weight density of the hydraulic fluid, $lb/ft^3$

$\gamma_m$ weight density of tube wall metal, $lb/ft^3$

$\eta_a$ efficiency of the pump, expressed as a decimal

$\eta_b$ efficiency of the motor, expressed as a decimal

$\eta_L$ efficiency of the hydraulic transmission lines, expressed as a decimal

$\nu$ factor in weight of pump and motor combination to account for the weight of accessories, such as scavenging pumps, supercharging pumps, filters, etc.

$\tau$ duration of power extraction, hr

$\Phi$ line density parameter, $lb/ft^3$, defined by Eq. (III-51)

$\psi$ specific thrust fuel consumption, lb/hr/lb

$\Omega$ flow coefficient, $ft^3/sec$, defined by Eq. (III-22)

## C. Derivation of the Non-Dimensional Pressure Drop Coefficient, $x^*$

The pressure drop in a fluid line is due to frictional losses and to losses in kinetic energy of the fluid at the several fittings and bends in the line. For purposes of this study, it is assumed that the kinetic energy losses will be considerably larger than the frictional losses. This assumes that there will be few long straight lengths of tubing, and that the Reynolds Number will be great enough to insure turbulent flow.

Under these conditions, the pressure loss in the lines may be expressed by

$$\Delta P = \frac{\gamma_f V^2}{2g}(K_1 + K_2 + K_3 + \ldots\ldots + K_n + f \frac{L}{D} \qquad \text{(III-1)}$$

where:

$\Delta P$ = total pressure loss in line, $lb/ft^2$

$\gamma_f$ = density of hydraulic fluid, $lb/ft^3$

$V$ = fluid velocity in line, ft/sec

$g$ = gravitational constant = 32.2 $ft/sec^2$

$K_1$, $K_2$, etc. = velocity head loss coefficient of each individual fitting, unit or bend in line (see Ref. III-2)

f = friction factor from Moody diagram

L = length of pressure line, ft

D = inside diameter of pressure line, ft

Let

$$\Sigma K = K_1 + K_2 + K_3 + \dots + K_n + f\frac{L}{D} \tag{III-2}$$

Then

$$\Delta P = \frac{\gamma_f V^2}{2g}\Sigma K \tag{III-3}$$

where:

$\Delta P$ = total pressure loss in the transmission lines

The flow in the lines is

$$Q = \frac{\pi}{4} V D^2 \tag{III-4}$$

Substituting the value of $V$ from Eq. (III-4) into Eq. (III-3)

$$\Delta P = \frac{16\ \gamma_f\ Q^2}{2g\pi^2\ D^4} \tag{III-5}$$

Let

P = pressure difference between the intake and discharge ports of the pump, (pump working pressure)

Then, dividing through by $P$ in Eq. (III-5)

$$\frac{\Delta P}{P} = \frac{8\ \gamma_f\ Q^2}{\pi^2\ g\ P\ D^4}\Sigma K \tag{III-6}$$

Let

$$G = \frac{8\ \gamma_f\ \Sigma K}{\pi^2\ g\ P} \tag{III-7}$$

and

$$x^{*2} = G\ \frac{Q^2}{D^4} \tag{III-8}$$

where

$$x^* = \text{pressure drop coefficient}$$

Then

$$\frac{\Delta P}{P} = x^{*2} \qquad \text{(III-9)}$$

D. Power Output of a Hydraulic System in Terms of the Pressure Drop Coefficient

A schematic diagram of a simple hydraulic power transmission system is shown in Fig. III-1. The pump, driven by the main engine, draws fluid from the reservoir, and forces it under pressure through a transmission line to the motor at some remote location in the aircraft. The motor converts the hydraulic energy into mechanical energy which drives an accessory gearbox. The fluid from the motor discharge port returns through a heat exchanger to the reservoir for de-aeration and recirculation.

Some energy is lost in the pump, the lines and the motor.

The power output of the hydraulic motor is expressed as:

$$HP = \frac{Q \ P_b \ \eta_b}{550} \qquad \text{(III-10)}$$

where:

$HP$ = shaft horsepower output of the motor

$Q$ = flow rate, $ft^3/sec$

$P_b$ = pressure differential between the ports of the motor, $lb/ft^2$

$\eta_b$ = efficiency of the motor

550 = conversion factor, ft lb/sec/hp

The working pressure at the motor is expressed by

$$P_b = P - \Delta P \qquad \text{(III-11)}$$

where:

$P$ = pressure difference between intake and discharge ports of the pump

[redacted]

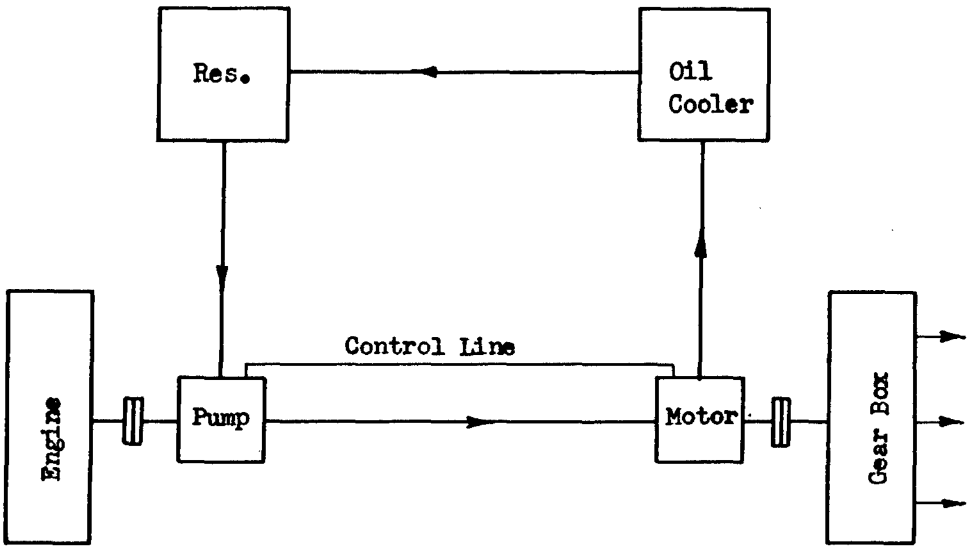



Fig. III-1 SCHEMATIC DIAGRAM OF SIMPLE HYDRAULIC POWER TRANSMISSION SYSTEM



[redacted]

Substituting the value of $P_b$, Eq. (III-11) into Eq. (III-10) yields:

$$HP = \frac{Q \eta_b}{550} P \left(1 - \frac{\Delta P}{P}\right) \tag{III-12}$$

The power output of the pump is expressed by:

$$HP_a = \frac{Q P}{550} = HP_{EXT} \eta_a \tag{III-13}$$

where:

$HP_a$ = power output of the pump

$HP_{EXT}$ = power extracted from engine power take-off shaft

$\eta_a$ = efficiency of the pump

Eq. (III-12) and Eq. (III-13) may be combined to give:

$$HP = HP_{EXT} \eta_a \eta_b \left(1 - \frac{\Delta P}{P}\right) \tag{III-14}$$

The efficiency of transmission is the ratio of the power output to the power input of the transmission line. Hence, Eq. (III-14) may be rearranged.

$$\frac{HP}{\eta_a \eta_b HP_{EXT}} = \left(1 - \frac{\Delta P}{P}\right) \tag{III-15}$$

Let

$$\eta_L = 1 - \frac{\Delta P}{P} \tag{III-16}$$

where:

$\eta_L$ = efficiency of fluid transmission

Then, Eq. (III-14) becomes

$$\frac{HP}{HP_{EXT}} = \eta_a \eta_b \eta_L \tag{III-17}$$

As shown by Eq. (III-9), the pressure loss coefficient is

$$\frac{\Delta P}{P} = x^{*2} \tag{III-18}$$

Substituting this value of $\frac{\Delta P}{P}$ into Eq. (III-16)

$$\eta_L = 1 - x^{*2} \tag{III-19}$$

The relationship between $\eta_L$ and $x^*$ is shown in Fig. III-2.

From Eq. (III-12) and Eq. (III-18)

$$HP = \frac{Q\ P}{550}\ \eta_b\ (1 - x^{*2}) \tag{III-20}$$

E. <u>Non-dimensional Relationship Between Flow Rate, Line Diameter, Flow Velocity and Pressure Loss Coefficient $x^*$</u>

From Eq. (III-20)

$$Q = \frac{550\ HP}{P\ \eta_b\ (1 - x^{*2})} \tag{III-21}$$

Let

$$\Omega = \frac{550\ \ HP}{P\ \eta_b} \quad (ft^3/sec) \tag{III-22}$$

Then

$$Q^* = \frac{Q}{\Omega} = \frac{1}{1 - x^{*2}} \tag{III-23}$$

where

$Q^*$ = the dimensionless flow

Fig. III-3 shows the variation of $Q^*$ with $x^*$ as expressed by Eq. (III-23).

From Eq. (III-8) and (III-9)

$$\frac{\Delta P}{P} = \frac{Q^2}{D^4}\ G \tag{III-24}$$

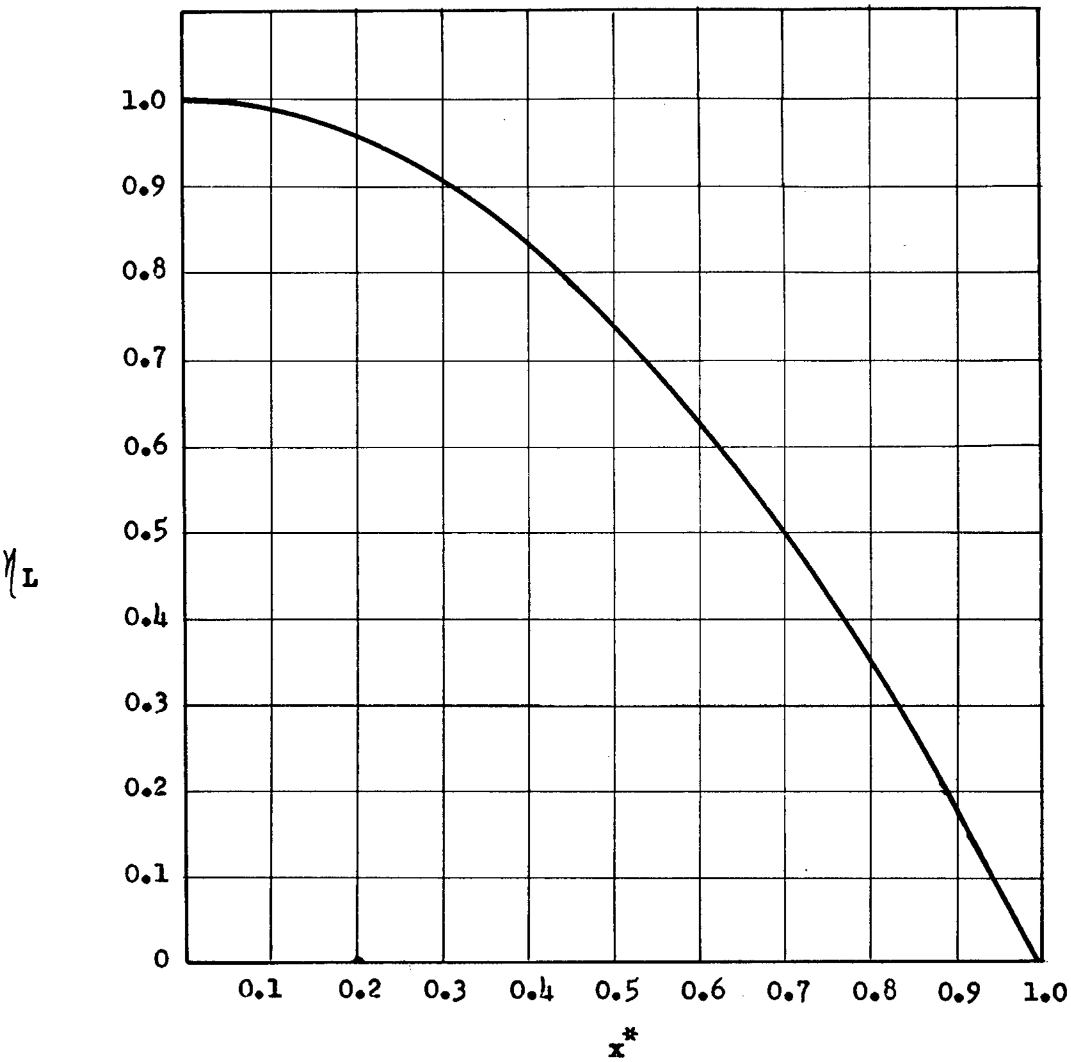



Fig. III-2 LINE EFFICIENCY AS A FUNCTION OF $x^*$ $\eta_L = 1 - x^{*2}$

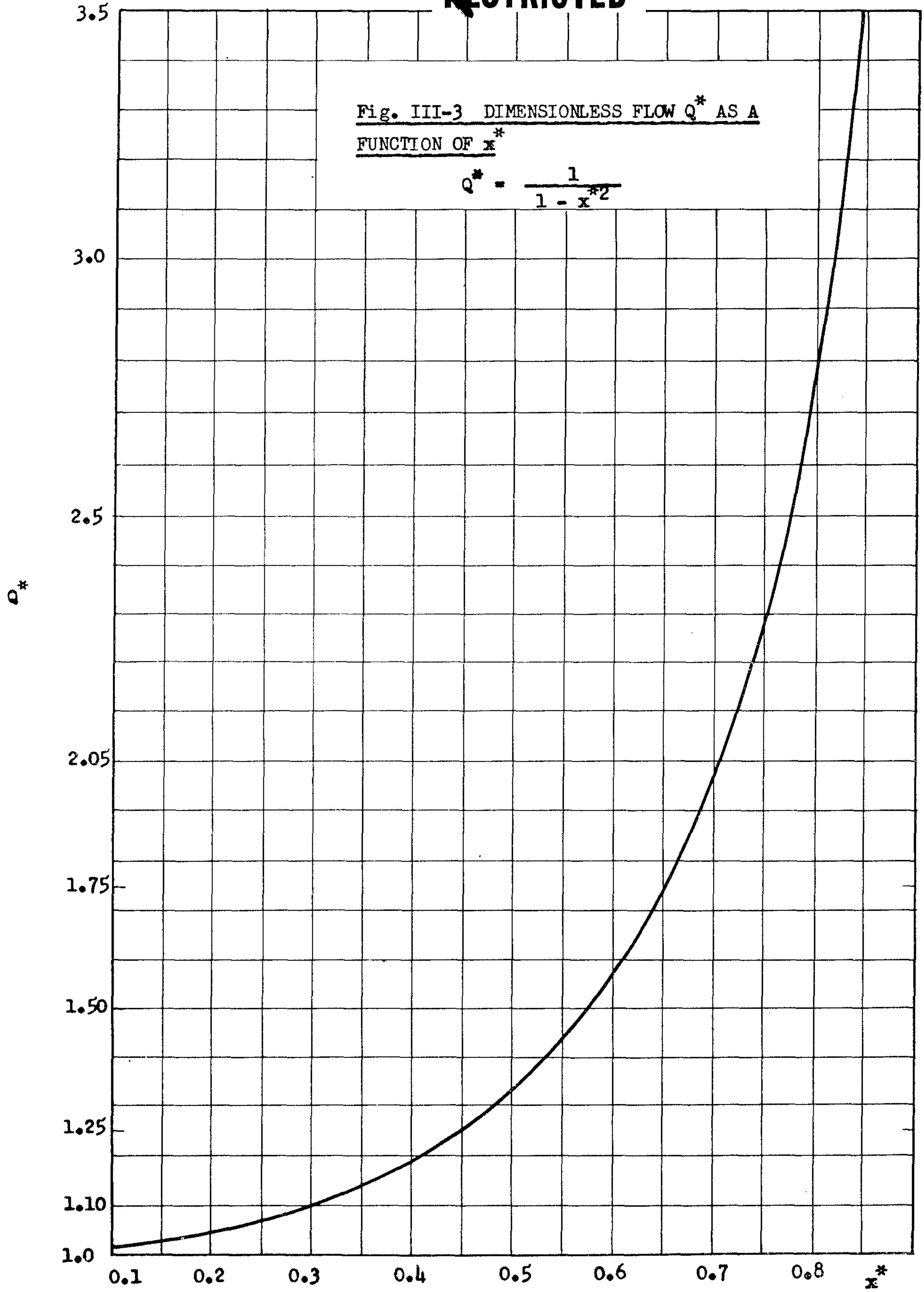



Fig. III-3 DIMENSIONLESS FLOW $Q^*$ AS A FUNCTION OF $x^*$

$$Q^* = \frac{1}{1 - x^{*2}}$$

Substituting the value of Q given by Eq. (III-23) and the value of $x^{*2}$ given by Eq. (III-18), Eq. (III-24) becomes

$$x^{*2} = \frac{\Omega^2 \quad G}{D^4 \quad (1 - x^{*2})^2} \tag{III-25}$$

$$D = \frac{\sqrt[4]{G}\,\sqrt{\Omega}}{\sqrt{x^{*} \; (1 - x^{*2})}} \tag{III-26}$$

Let

$$D^{*} = \frac{D}{\sqrt[4]{G}\,\sqrt{\Omega}} = \frac{1}{\sqrt{x^{*} \; (1 - x^{*2})}} \tag{III-27}$$

where

$D^{*}$ is a dimensionless diameter

Eq. (III-27) is shown graphically in Fig. III-4.

The velocity of fluid in the line can be found from

$$V = \frac{Q}{a} = \frac{4}{D^2}\,\frac{Q}{\pi} \tag{III-28}$$

where:

$V$ = velocity of fluid in the transmission line, ft/sec

$a$ = flow area of the transmission line, $ft^2$

Substituting the values of Q and D from Eqs. (III-23) and (III-26) respectively into Eq. (III-28) yields

$$V = \frac{4 \; \Omega \; x^{*}}{(1 - x^{*2})\;\pi}\;\frac{(1 - x^{*2})}{G\;\Omega} = \frac{4x^{*}}{\pi\sqrt{G}} \tag{III-29}$$

Let

$$V^{*} = \sqrt{G} \quad V = \frac{4}{\pi}\,x^{*} \tag{III-30}$$

where $V^{*}$ is a dimensionless velocity.

[redacted]

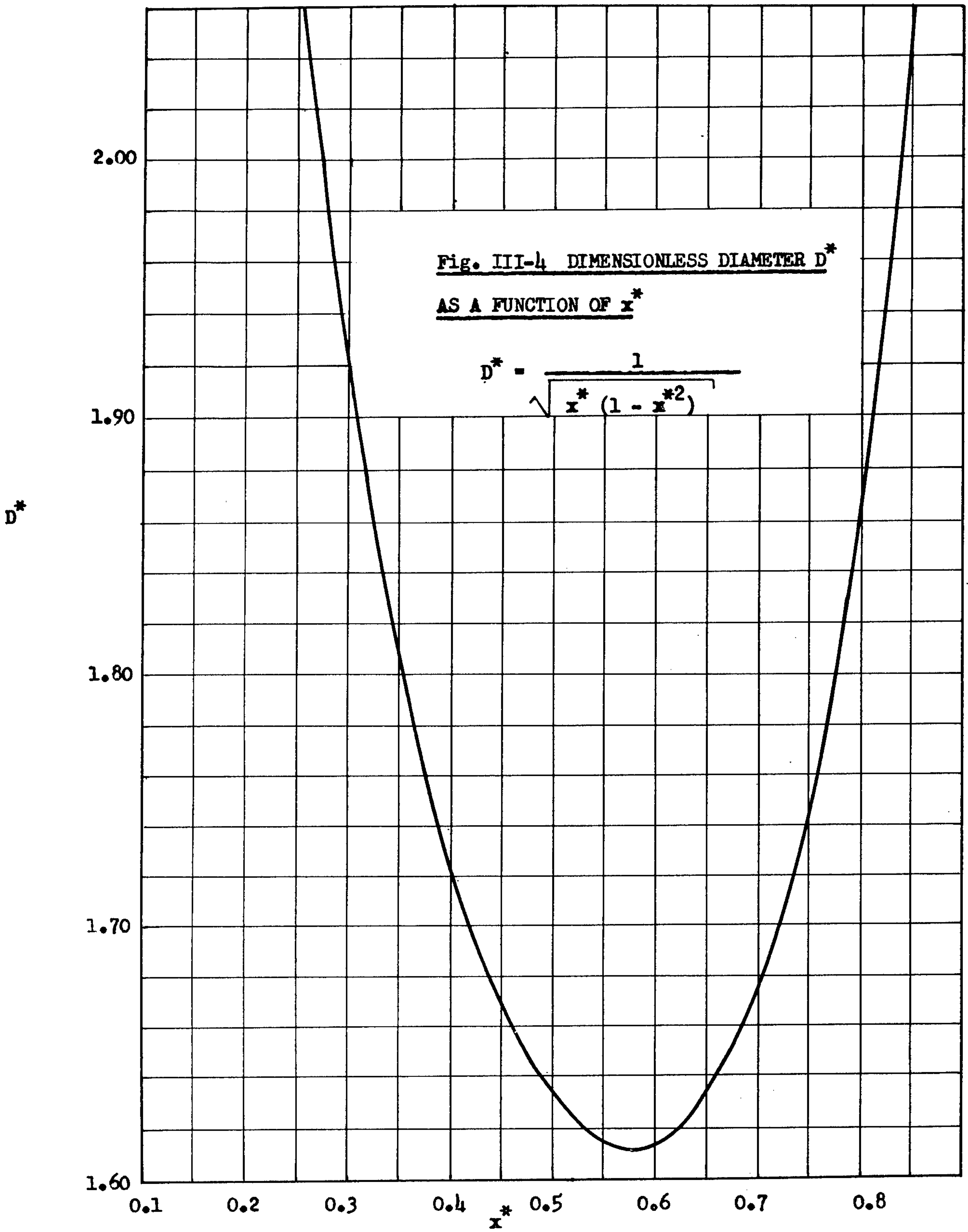



Fig. III-4 DIMENSIONLESS DIAMETER $D^*$ AS A FUNCTION OF $x^*$

$$D^* = \frac{1}{\sqrt{x^* (1 - x^{*2})}}$$



[redacted]

## F. Power Characteristics of Hydraulic Systems

The power requirements of aircraft accessory drives may be quite variable. The maximum power required may be 200 per cent or more of the normal output of the system. The characteristics of the system must be carefully determined in order that the system be adequate, but not overdesigned.

For a given system, the power output is a function of the flow, pressure and transmission line losses. If one of these properties is held constant, the power output is then determined by the other two variables. However, the transmission line losses can be expressed, by means of the pressure drop coefficient $x^*$, as a function of either the pump working pressure or the system flow. The system output can, therefore, be expressed in terms of one variable.

In the following, the relationship between the power output and the system variable is derived for the two systems considered in this report. The two systems are named from these relationships. They are

(a) The constant flow, variable pressure system

(b) The constant pressure, variable flow system

### 1. Power Characteristics of a Constant Flow, Variable Pressure System

The constant flow, variable pressure, hydraulic power transmission system utilizes a variable displacement pump and a constant displacement motor. If the engine speed is reduced, the pump displacement is increased so that the flow remains constant. The power output of the motor is determined by the pressure. As the load increases, the system pressure increases. The maximum power output of the system depends upon the maximum permissible pressure.

For short durations, the system pressure may be allowed to rise above the rated pressure. The constant flow, variable pressure system is, therefore, capable of transmitting short duration loads in excess of its rated power.

Eq. (III-21) shows the relationship between the power output, pump pressure, flow and the independent variable $x^*$ for a hydraulic power transmission system, as:

$$\frac{550 \quad HP}{Q\, P\, \eta_b} = 1 - x^{*2} = 1 - \frac{\Delta P}{P} \qquad \text{(III-31)}$$

This equation may be written as:

$$\frac{550\ HP}{Q\ \eta_b} = P - \Delta P \qquad \text{(III-32)}$$

[redacted]

The maximum power is expressed by

$$\frac{550}{Q\,\eta_b}\,HP_m = P_m - \Delta P \quad \text{(III-33)}$$

where

$HP_m$ = maximum power output

$P_m$ = maximum permissible operating pressure of units

Subtracting Eq. (III-32) from Eq. (III-33)

$$\frac{550}{Q\,\eta_b}\,(HP_m - HP) = P_m - P \quad \text{(III-34)}$$

or,

$$\frac{550}{Q\,\eta_b} = \frac{P_m - P}{HP_m - HP} \quad \text{(III-35)}$$

Substituting Eq. (III-35) into Eq. (III-32) results in:

$$\frac{P_m - P}{HP_m - HP}\,HP = P - \Delta P \quad \text{(III-36)}$$

Solving for P gives:

$$P = \Delta P + \frac{P_m - \Delta P}{HP_m}\,HP \quad \text{(III-37)}$$

Eq. (III-37) shows that the pump pressure is a straight line function of the power output. With a constant flow the slope of this line is dependent upon the pressure drop, and a pre-determined operating point. In the preceding analysis, this point has been chosen to correspond to maximum power at maximum permissible pressure. If no specific overload is required, the operating point may be chosen to correspond to the rated power required by the accessories, with the system at the rated pressure of the hydraulic units.

A schematic diagram illustrating the pressure-power relationship for the two cases is shown in Fig. III-5. The maximum permissible pressure of the system is assumed to be 150 per cent of the rated pressure. Line AB represents a system designed for 200 per cent power at maximum pressure. Line CD represents a system designed for rated power at rated pressure. $HP_{cr}$, the cruise



[redacted]

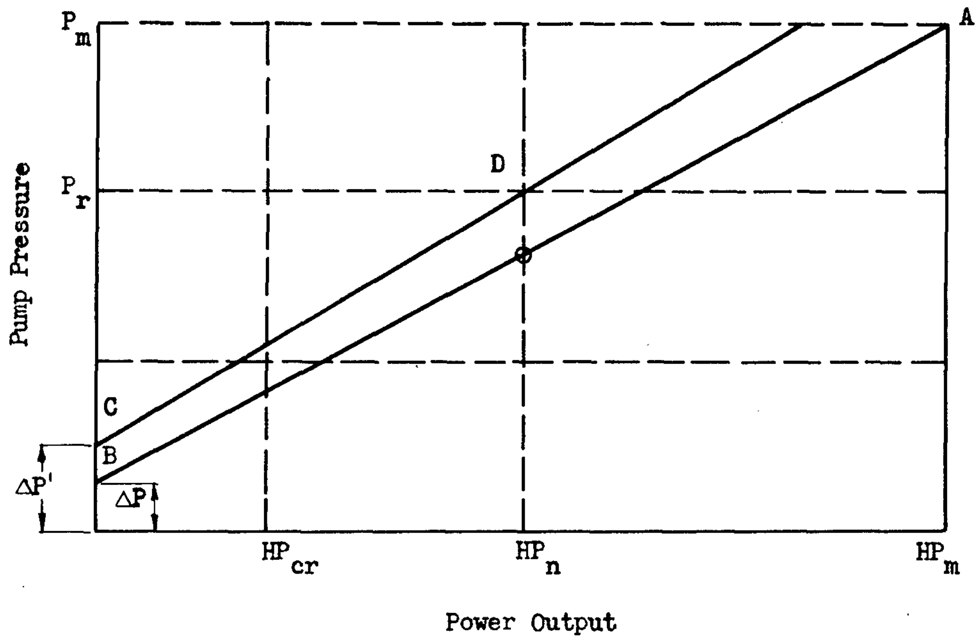



Fig. III-5 GENERAL RELATIONSHIP BETWEEN PRESSURE AND POWER OUTPUT OF A CONSTANT FLOW VARIABLE PRESSURE HYDRAULIC POWER TRANSMISSION SYSTEM

power requirements of the accessory system, may be any fraction of the normal rated power HP. When $HP_{cr} = 0$, then $P = \Delta P$.

2. Power Characteristics of a Constant Pressure, Variable Flow System

The constant pressure, variable flow hydraulic power transmission system utilizes variable displacement units for both pump and motor. The system tends to operate at a constant pressure under all load conditions.

Since the pump working pressure is to be held constant regardless of the load, the line must be large enough to handle the flow required at maximum load.

For a constant pump output pressure, the relationship between flow and power may be determined as follows:

From Eq. (III-27)

$$\Omega = \frac{D^2}{\sqrt{G}\; D^{*2}} = \frac{D^2\, x^* \,(1 - x^{*2})}{\sqrt{G}} \qquad \text{(III-38)}$$

From Eq. (III-22), however:

$$\Omega = \frac{550 \;\; HP}{P \;\eta_b} \qquad \text{(III-39)}$$

If Eq. (III-38) and Eq. (III-39) are combined, the following result is obtained:

$$\frac{550 \;\; HP \sqrt{G}}{P \;\eta_b \;\; D^2} = x^* \,(1 - x^{*2}) \qquad \text{(III-40)}$$

The above expression is plotted against $x^*$ in Fig. III-6. From Eq. (III-8)

$$x^* = \sqrt{G} \; \frac{Q}{D^2} \qquad \text{(III-41)}$$

Therefore, Fig. 6 represents the power available from a line of a given diameter and configuration at any flow rate. As the flow rate increases the power increases. Since the transmission line losses are proportional to $Q^2$, these losses become increasingly important as the flow increases. Thus, the horsepower reaches a maximum at $x^* = 0.577$ and then decreases. If the power required from this sytem lies above the curve, it is necessary to increase the line size, D, or the pump pressure, P.

[redacted]

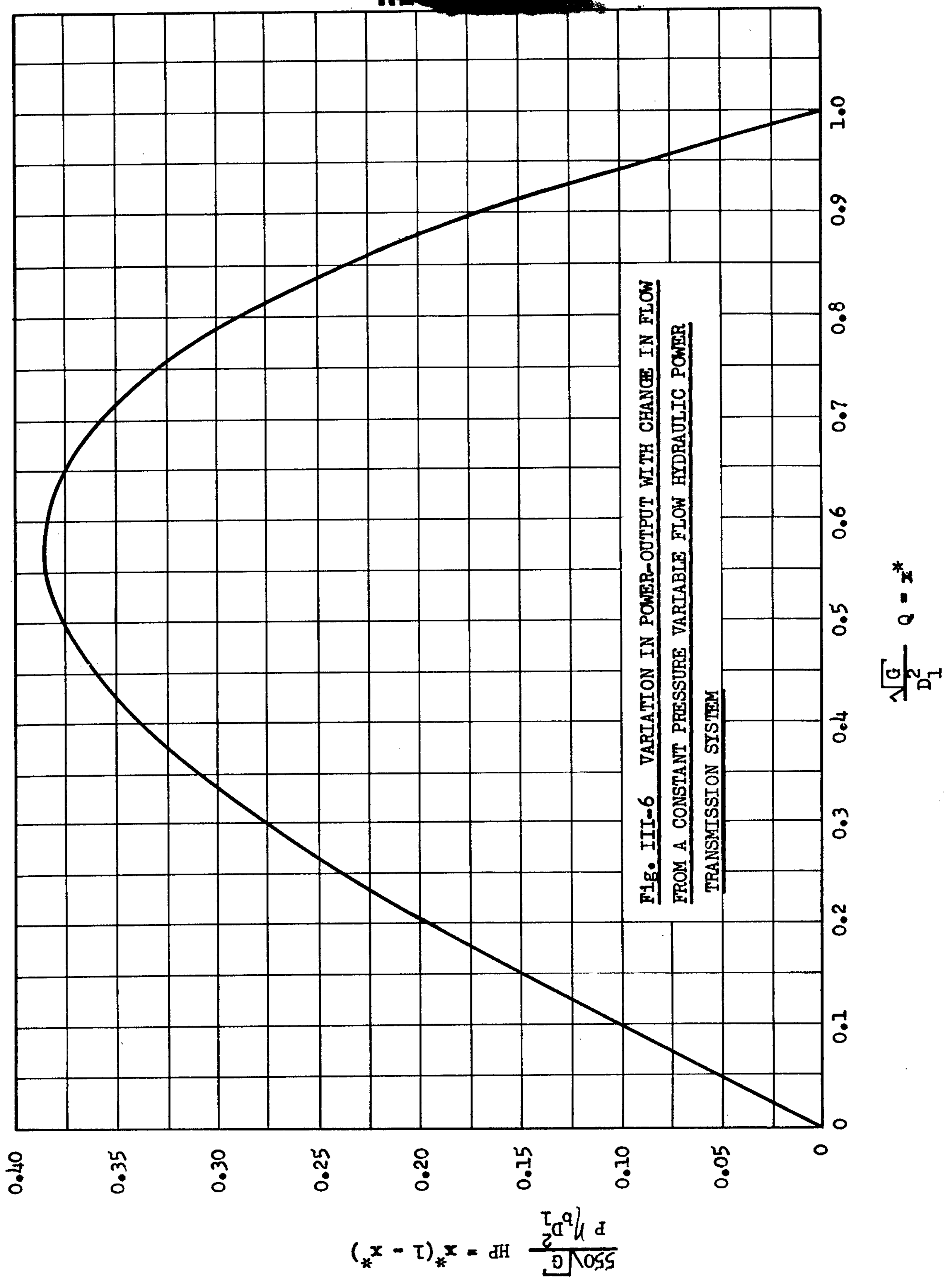



Fig. III-6 VARIATION IN POWER-OUTPUT WITH CHANGE IN FLOW FROM A CONSTANT PRESSURE VARIABLE FLOW HYDRAULIC POWER TRANSMISSION SYSTEM



[redacted]

## G. Weight of Hydraulic Transmission Lines

This analysis of the hydraulic transmission lines is based on the assumption that the flow of fluid in the system may be reversible. Therefore, all of the transmission lines may be subjected to high pressure, and the tube walls must be of sufficient thickness to withstand the pressure stresses.

### 1. Cross-Sectional Area of Metal in a Tube

The area of the metal in a cross-section of a tube is given by

$$A_m = \frac{\pi (D_o^2 - D^2)}{4} \qquad \text{(III-42)}$$

where:

$A_m$ = area of metal, $ft^2$

$D_o$ = outside diameter, ft

$D$ = inside diameter, ft

The outside diameter, $D_o$, can be written as

$$D_o = D + 2t \qquad \text{(III-43)}$$

where

$t$ = wall thickness of the tube

From the well known equation for thinwalled cylinders, wall thickness can be written as

$$t = \frac{P_s D}{2s} \qquad \text{(III-44)}$$

where:

$P_s$ = maximum pressure to which the lines may be subjected due to pump working pressure and any supercharge pressure. This pressure is a constant of the lines.

$s$ = the maximum permissible working stress, $lb/ft^2$

Substituting this value of $t$ in Eq. (III-42) gives

$$A_m = \pi \left[ \frac{D^2 P_s}{2s} + \frac{D^2 P_s^2}{4s^2} \right] = \frac{P_s D^2 \pi}{2s} \left[ 1 + \frac{P_s}{2s} \right] \qquad \text{(III-45)}$$

2. Line Weight

The weight of the tube is given by

$$W_m = A_m \ L \ \gamma_m \tag{III-46}$$

where:

$W_m$ = weight of the tube, lb

$\gamma_m$ = density of the line material, lb/ft$^3$

$L$ = length of the line, ft

Combining Eq. (III-45) and Eq. (III-46) yields,

$$W_m = L \ \gamma_m \ \pi \ \frac{D^2 \ P_s}{2s} \ (1 + \frac{P_s}{2s}) \tag{III-47}$$

The weight of the fluid in the line is

$$W_l = a \ L \ \gamma_f = \frac{\pi D^2}{4} \ \gamma_f \ L \tag{III-48}$$

where:

$a$ = flow area of the tube

The total weight of line and fluid is obtained by adding Eq. (III-47) and Eq. (III-48).

$$W_L = \frac{\pi \ L \ \gamma_m \ P_s \ D^2}{(2s)} \left[ 1 + \frac{P_s}{2s} \right] + \frac{\pi D^2 \gamma_f \ L}{4} \tag{III-49}$$

where:

$W_L$ = combined weight of line and fluid

This equation may be further modified to reflect the weight of tube clamps and fittings, by adding a factor m to the density of the metal. The factor m should be based upon analysis of the projected system, or upon experience and statistical data. Eq. (III-49) then becomes

$$W_L = \frac{\pi P_s \ \gamma_m}{(2s)} \ (1 + m) \ \ (1 + \frac{P_s}{2s}) + \frac{\pi \ \gamma_f}{4} \ L \ D^2 \tag{III-50}$$

Let

$$\Phi = \frac{\pi P_s \gamma_m (1 + m) \left(1 + \frac{P_s}{2s}\right)}{2s} + \frac{\pi \gamma_f}{4} \ (\mathrm{lb/ft^3}) \qquad \text{(III-51)}$$

then

$$W_L = \Phi L D^2 \qquad \text{(III-52)}$$

Incorporating Eq. (III-26) into Eq. (III-52) gives

$$W_L = \frac{\Phi L \sqrt{G} \, \Omega}{x^* (1 - x^{*2})} \qquad \text{(III-53)}$$

H. <u>Weight of Reservoir</u>

The reservoir capacity is assumed to be dependent upon the line volume and upon the duration of power extraction. The volume of the reservoir may thus be written:

$$V_r = \alpha V_L + \beta V_L \tau \qquad \text{(III-54)}$$

where:

$V_r$ = volume of the reservoir, $ft^3$

$V_L$ = volume of the line, $ft^3$

$\alpha$ = fraction of line volume to be carried in reservoir for fluid de-aeration, and contraction purposes

$\beta$ = fraction of line volume to be carried in reservoir for each hour of power extraction to compensate for small seepage leaks.

$\tau$ = duration of power extraction, hrs

but,

$$V_L = D^2 \frac{\pi}{4} L \qquad \text{(III-55)}$$

Therefore,

$$V_r = \frac{\pi}{4} L (\alpha + \beta \tau) D^2 \qquad \text{(III-56)}$$

The weight of the fluid in the reservoir is

$$W_{fr} = \frac{\pi}{4} (L) \ (\alpha + \beta \tau) \ \gamma_f \ D^2 \qquad \text{(III-57)}$$

The weight of the reservoir may thus be written as

$$W_R = \frac{\pi}{4} L \left(\alpha + \beta \tau\right) \gamma_f \, h \, D^2 \tag{III-58}$$

where:

h = weight factor for material required to enclose a unit volume of fluid, dependent upon material, wall thickness and shape of reservoir. This factor should include an allowance for fluid aeration volume in the reservoir.

Combining Eq. (III-58) and Eq. (III-27)

$$W_R = \frac{\pi}{4} \frac{L \left(\alpha + \beta \tau\right) \sqrt{G}\, \Omega \gamma_f \, h}{x^* \left(1 - x^{*2}\right)} \tag{III-59}$$

I. Weight of Fuel Consumed by a Hydraulic Power Transmission System

Included in the total weight of an accessory system is the weight of fuel required to operate the accessories throughout the flight of the airplane. The specific fuel consumption $C''_{PX}$ indicates the amount of additional fuel a given engine consumes to supply a unit of power to the accessory transmission system for one hour while providing the required thrust.

The specific fuel consumption of the transmission system is defined by:

$$C''_{PX} = \frac{W_F}{HP_{EXT} \, \tau} \tag{III-60}$$

where:

$W_F$ = weight of fuel required to operate the accessory system for time, $\tau$, lb

$C''_{PX}$ = specific fuel consumption, lb of fuel per HP hr extracted

$HP_{EXT}$ = power extracted from the airplane engine, HP

$\tau$ = duration of power extraction, hr

It is desired to develop an expression for $C''_{PX}$ in terms of engine operating conditions which can be calculated from available data.

The effect of power extraction on the fuel consumption and thrust of the jet engine is shown schematically in Fig. III-7. When power is extracted,

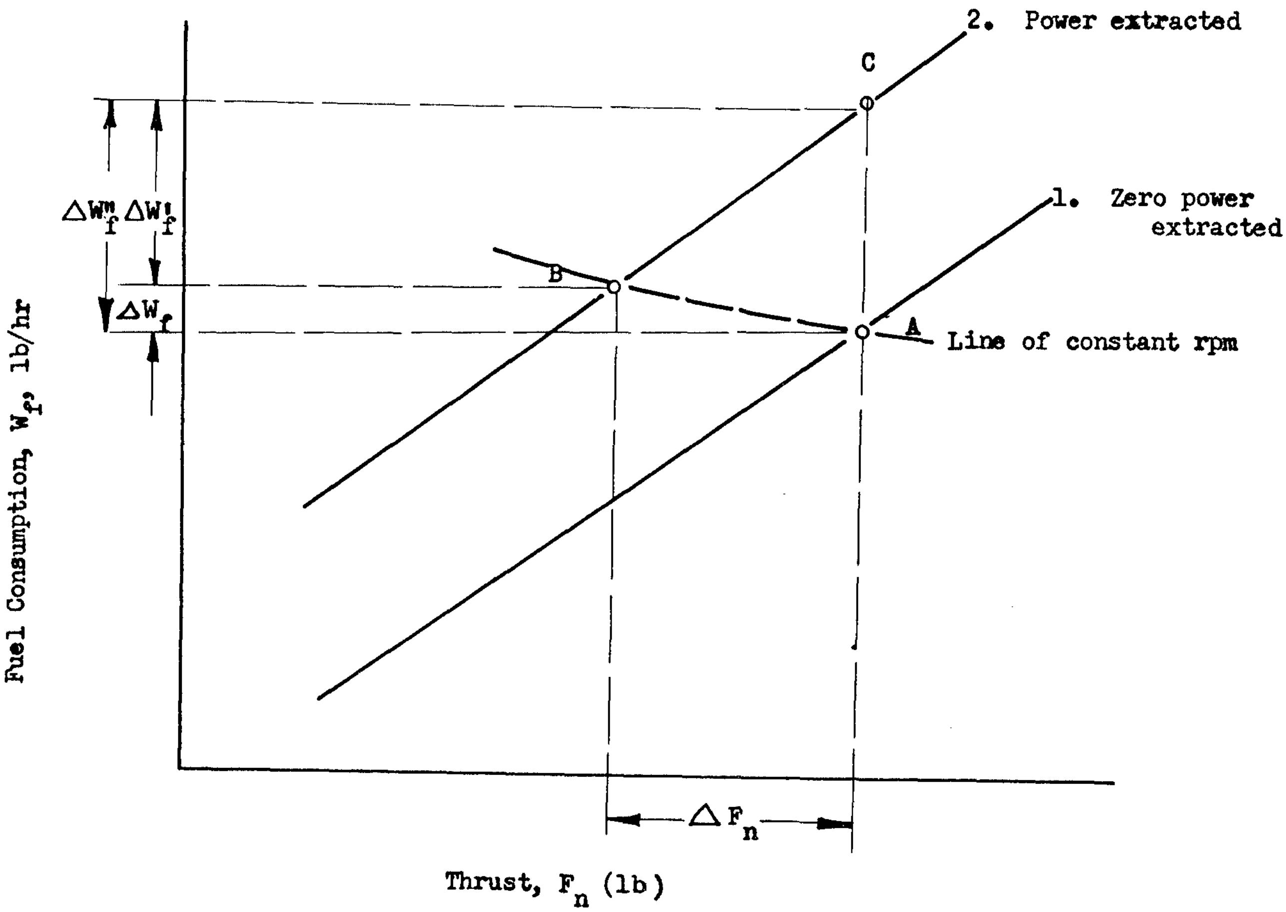



Fig. III-7 EFFECT OF ACCESSORY POWER EXTRACTION ON ENGINE PERFORMANCE

the engine speed tends to decrease. However, since the engine control is primarily a speed sensing device, it acts to return the engine rpm to its initial value by increasing the fuel flow (point A to point B). The thrust of the engine is also decreased because the turbine is using a larger percentage of the available energy to drive the compressor and the accessories. The power loss may be recovered by increasing the setting of the power control lever in the aircraft. This raises the base speed of the engine control and results in a further increase in fuel consumption (point B to point C).

Although not in accordance with Mil-E-5008, the following notation will be used in this report.

The changes in thrust and fuel consumption along a constant rpm line due to power extraction are:

$$\Delta F_n = C_{PX} \left[ \frac{HP_{EXT}}{HP_{REF}} \right] F_n \qquad \text{(III-61)}$$

and

$$\Delta W_f = C'_{PX} \left[ \frac{HP_{EXT}}{HP_{REF}} \right] W_f \qquad \text{(III-62)}$$

where

$C_{PX}$, $C'_{PX}$ = power extraction correction factors for thrust and fuel consumption respectively

$\Delta F_n$ = change in engine thrust, lb

$HP_{EXT}$ = total power extracted from the engine by the power transmission system

$HP_{REF}$ = a reference power used for comparison of jet engine performance at any operating condition and arbitrarily chosen as 10 per cent of the jet power at sea level static conditions

$\Delta W_f$ = change in fuel consumption, lb/hr

$W_f$ = fuel consumption, lb/hr

From the engine specifications the change of net thrust is known for a given change of fuel flow. Fig. III-7 shows a plot of fuel consumption versus net thrust for a constant flight speed and an unburdened engine. In the normal operating range of the engine this plot is nearly a straight line. The slope of the line is given by:

$$\psi = \frac{d\,W_f}{d\,F_n} \qquad \text{(III-63)}$$

The change in fuel flow for a given change in thrust along a constant flight speed line is given by:

$$\Delta W'_f = \psi \Delta F_n \tag{III-64}$$

where:

$\Delta W'_f$ = change in fuel consumption, lb/hr

The net increase in fuel consumption due to extracting power from the engine while maintaining a constant thrust, as shown in Fig. III-7, is given by

$$\Delta W''_f = \Delta W_f + \Delta W'_f \tag{III-65}$$

where

$\Delta W''_f$ = total change in fuel flow due to power extraction, lb/hr

With consideration of Eqs. (III-61), (III-62), (III-64), this can be written as:

$$\Delta W''_f = C'_{PX} \, W_f \, \frac{HP_{EXT}}{HP_{REF}} + \psi C_{PX} \, F_n \, \frac{HP_{EXT}}{HP_{REF}} \tag{III-66}$$

or

$$\Delta W''_f = \left( \frac{C'_{PX} \, W_f + \psi C_{PX} \, F_n}{HP_{REF}} \right) HP_{EXT} \tag{III-67}$$

Let

$$C''_{PX} = \frac{C'_{PX} \, W_f + \psi C_{PX} \, F_n}{HP_{REF}} \tag{III-68}$$

Then

$$\Delta W''_f = C''_{PX} \; HP_{EXT} \tag{III-69}$$

or, in terms of the power output of the hydraulic transmission system at cruise conditions

$$\Delta W''_f = \frac{C''_{PX} \; HP_{cr}}{\eta_a \, \eta_b \, \eta_L} \tag{III-70}$$

The coefficient $C''_{PX}$ as defined by Eq. (III-67) is called the specific fuel consumption chargeable to the accessories and is determined from engine specifications, for any given operating condition of the engine.

Multiplying Eq. (III-69) by the time of flight gives the approximate weight of fuel consumed by the **engine.**

$$W_F = \Delta W''_f \tau = \frac{C''_{PX} \tau HP_{cr}}{\eta_a \eta_b \eta_L} \tag{III-71}$$

where:

$\tau$ = duration of accessory drive operation, hr

$HP_{cr}$ = cruise power output of accessory drive, HP

J. Weight of a Pump and Motor Set

The weight of a hydraulic pump or motor, if similarity of design is assumed, has been found to vary in accordance with the general equation:

$$\frac{W_a}{P_r^{1/3}} = b\left(\frac{HP}{N_r \; P_o}\right)^{2/3} + c \tag{III-72}$$

where

$W_a$ = weight of unit

b = constant

HP = power output for units operating at Pressure P

$N_r$ = rated speed

$P_r$ = normal rated pressure, $lb/in^2$

$P_o$ = operating pressure, $lbs/in^2$

c = constant

Fig. III-8 shows this correlation by plotting the values of $W/P_r^{1/3}$ against $HP_r/N \; P_r$ for hydraulic pumps per Ref. III-5 for $P_o = P_r$. The slope of the curve is 2/3.

Due to differences in the basic philosophy of design, the constants, B and C, will vary for each basic type of pump or motor design. These constants will be found for one basic type of pump and motor, assuming the following design conditions:

Design rated pressure of pump 2200 psi

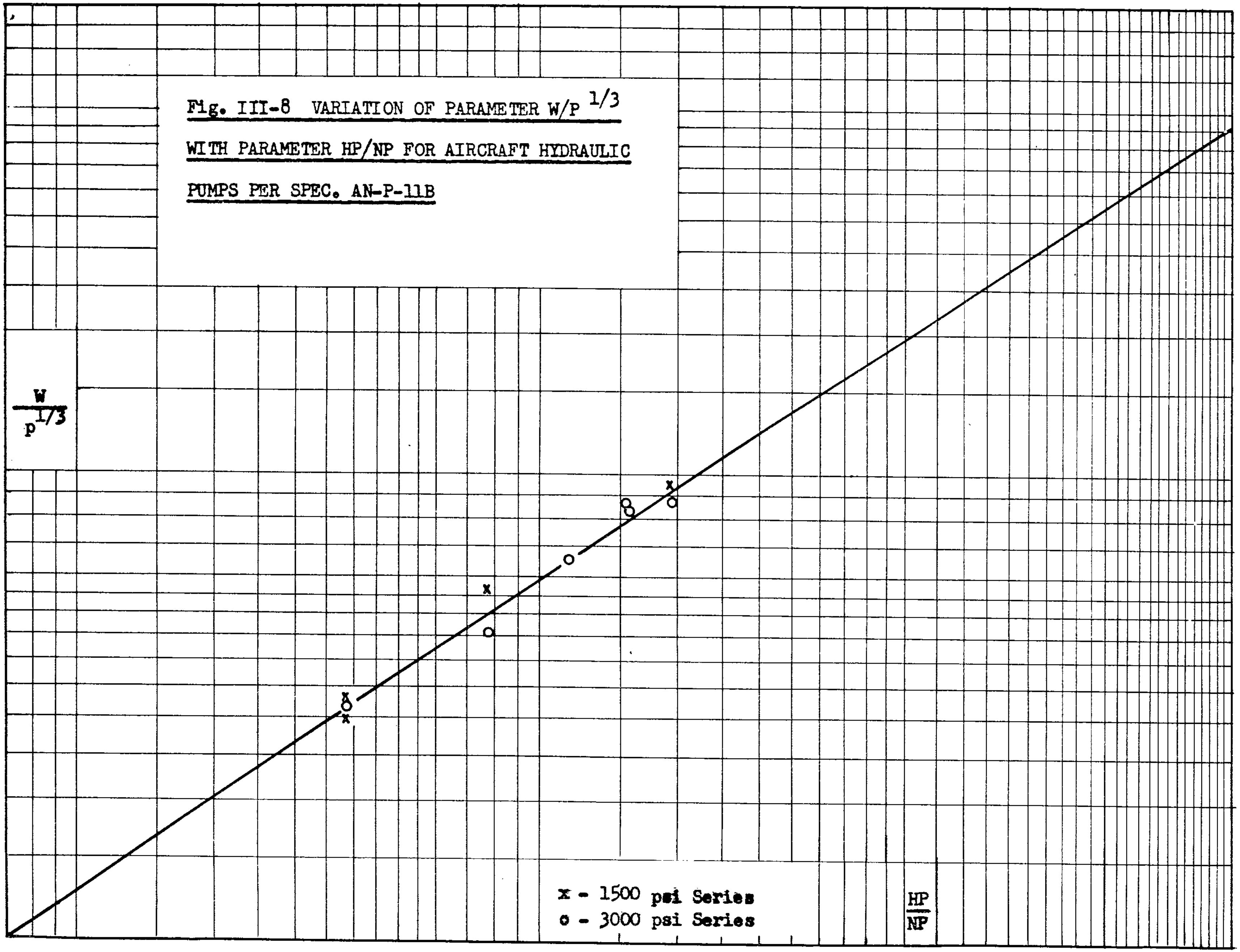



Fig. III-8 VARIATION OF PARAMETER $W/P^{1/3}$ WITH PARAMETER HP/NP FOR AIRCRAFT HYDRAULIC PUMPS PER SPEC. AN-P-11B

| | |
|---|---|
| Design rated speed of pump | $N_a$ = 3000 rpm |
| Design rated pressure of motor | 2200 psi |
| Design rated speed of motor | $N_b$ = 6000 rpm |

The design rated speed $N_a$ of the pump is defined as that minimum speed at which the rated capacity can be maintained. If the two units are considered as close coupled, it is apparent that the equivalent hydraulic horsepower of the pump must be greater than the output power of the motor by the inefficiency of the motor. Thus:

$$HP_a = \frac{HP_b}{\eta_b} \tag{III-73}$$

In order to rate the pump at an equivalent parametric value, the ratio $HP/N\,P_o$ for the pump must be expressed at the same speed as that of the motor. Since $N_a = 1/2\ N_b$, the parameter may be expressed as:

$$\frac{HP_a}{N_a\ P_o} = \frac{2 \cdot HP_b}{\eta_b\ N_b\ P_o} \tag{III-74}$$

$N_b$ then can be defined as the rated speed, $N_r$, of the motor-pump combination. For the combination of pump and motor, the weight may be expressed as:

$$\frac{W_u}{P_r^{\ 1/3}} = \left[\left(\frac{2}{\eta_b}\right)^{2/3} + 1\right] b \left(\frac{HP_b}{N_b\ P_o}\right)^{2/3} + c \tag{III-75}$$

where:

$P_r$ = rated pressure of the units

$P_o$ = operating pressure

Assuming a motor efficiency of 0.9,

$$\frac{W_u}{P_r^{\ 1/3}} = 2.703\ b \left(\frac{HP_b}{N_r\ P_o}\right)^{2/3} + c \tag{III-76}$$

For the units plotted on Fig. III-9, Eq. (III-76) becomes:

$$\frac{W_u}{P_r^{1/3}} = 16{,}000 \left( \frac{HP_b}{N_r\, P_o} \right)^{2/3} + 18 \qquad \text{(III-77)}$$

Introducing a factor $\nu$ to account for the weight of supercharging pumps, scavenging pumps, etc., which are not found on the units represented by (Spec. AN-P-11b) the above equation may be written as:

$$\frac{W_u}{P_r^{1/3}} = 16{,}000 \left( \frac{HP_b}{N_r\, P_o} \right)^{2/3} (1 + \nu) + 18 \qquad \text{(III-78)}$$

The factor $\nu$ can be evaluated for a given pump and motor design if the weights of the units, the power output, and the corresponding pressure and speed are known. From published data, the value of $\nu$ has been evaluated for one pump and motor combination. This value was determined to be $\nu = 0.22$.

Eq. (III-78) may be written as:

$$W = 16{,}000\, P_r^{1/3} \left( \frac{HP_b}{N_r\, P_o} \right)^{2/3} (1 + \nu) + 18 \qquad \text{(III-79)}$$

Since $HP/N_r\, P_o$ is equal to the displacement, it may be seen that the weight is a function of the normal rated pressure and the displacement. For the calculated value of $\nu = 0.22$, Eq. (III-79) becomes:

$$W = 19{,}500 \left[ \frac{HP_b^2}{\frac{N_r^2\, P_o^2}{P_r}} \right]^{1/3} + 18 \qquad \text{(III-80)}$$

This equation is shown graphically by Fig. III-9. Two curves are shown for the weight power relationship. The upper curve corresponds to the normal rated power; the lower curve to the maximum power available for short time intervals.

Eq. (III-80) expresses the weight of the pump-motor combination when the pressures are expressed in $lbs/in^2$. To maintain consistence of units throughout the report, the equation may be changed to

$$W = 102{,}200 \left[ \frac{HP_b^2}{\frac{N_r^2\, P_o^2}{P_r}} \right]^{1/3} + 18 \qquad \text{(III-81)}$$

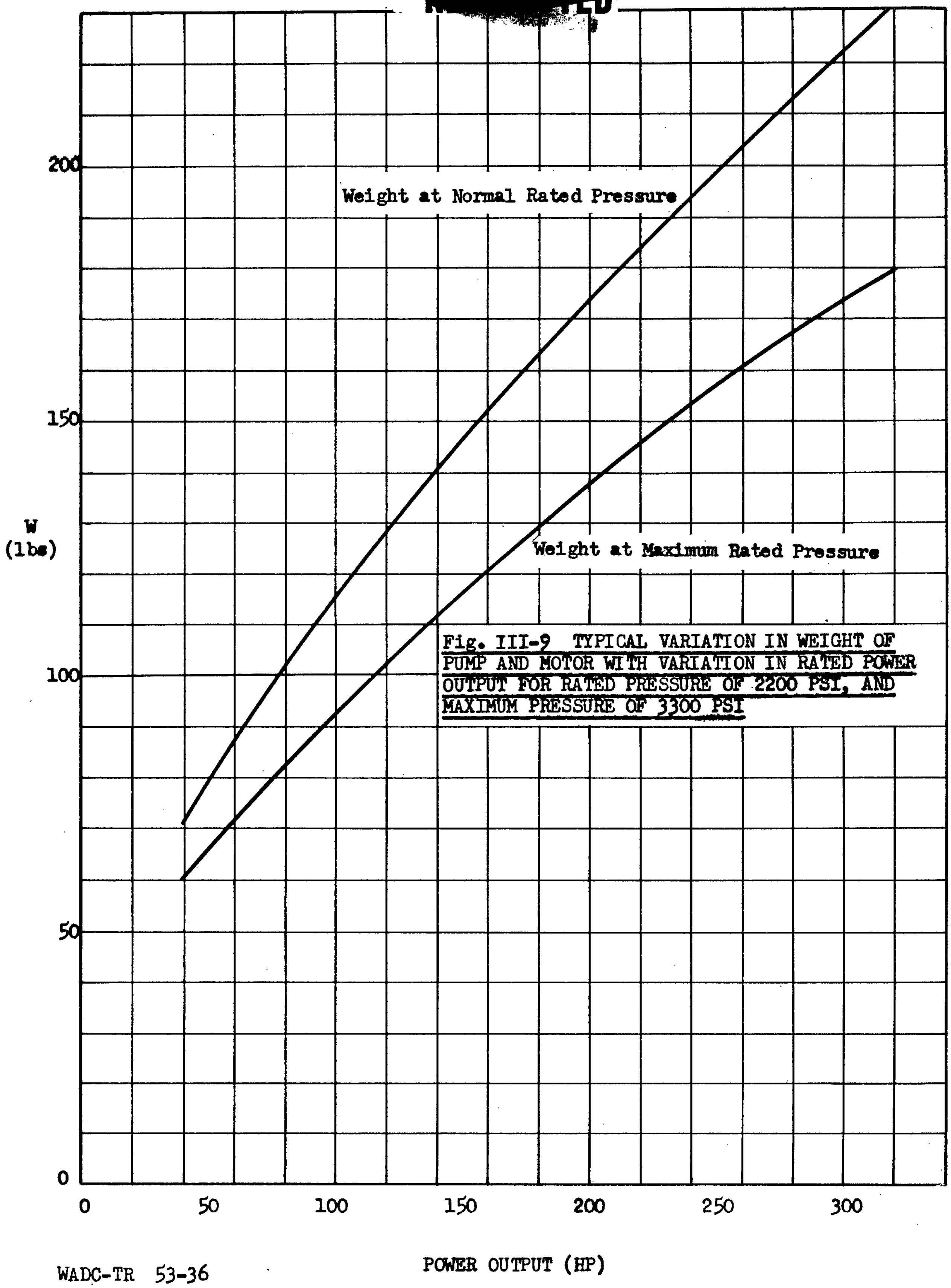



Fig. III-9 TYPICAL VARIATION IN WEIGHT OF PUMP AND MOTOR WITH VARIATION IN RATED POWER OUTPUT FOR RATED PRESSURE OF 2200 PSI, AND MAXIMUM PRESSURE OF 3300 PSI

where

$$P_o \text{ and } P_r = \text{pressure in lbs/ft}^2$$

The rate of change in weight of the units with change in power output may be obtained by differentiating Eq. (III-81) and gives

$$\frac{dW_s}{d\,HP_b} = \frac{68,140}{N_r^{2/3}\left(\frac{P_o^2}{P_r}\right)^{1/3} HP_b^{1/3}} \qquad \text{(III-82)}$$

K. <u>Weight of Pump and Motor Set for a Hydraulic Power Transmission System</u>

The weight of the pump and motor combination may be expressed as a continuous function of the power output.

If the units are close coupled; that is, if the line losses are assumed to be zero, the net power output of the system is equal to the rating of the pump and motor combination. If the units are connected by a transmission line, the power required to compensate for the line losses must be added to the required power output. Therefore, units of larger capacity must be used.

The increment in power rating of the units can be determined as follows:

From Eq. (III-17)

$$HP_{EXT}\ \eta_a\ \eta_b = \frac{HP}{\eta_L} \qquad \text{(III-83)}$$

Substituting the value of transmission line efficiency as shown in Eq. (III-16)

$$HP_{EXT}\ \eta_a\ \eta_b = \frac{HP}{1 - x^{*2}} = HP\left(1 + \frac{x^{*2}}{1 - x^{*2}}\right) \qquad \text{(III-84)}$$

The line power losses in Eq. (III-84) are represented by the portion:

$$\Delta HP_L = HP\ \frac{x^{*2}}{1 - x^{*2}} \qquad \text{(III-85)}$$

where:

$$\Delta HP_L = \text{line power losses}$$

1. <u>Weight of Pump and Motor for a Constant Flow, Variable Pressure System</u>

The weight of the units can now be expressed in terms of the net power output and the line losses

$$W_u = W_s + \Delta W_s \qquad \text{(III-86)}$$

where:

$W_u$ = weight of required pump-motor combination

$W_s$ = weight of pump and motor having power output equal to the net output of the system

$\Delta W_s$ = weight of pump-motor increment required for overcoming line losses

The value of $\Delta W_s$ can be approximated by $(dW_s/dHP)\ \Delta HP$, since for increments of $\Delta HP$, the weight-power output curve is nearly a straight line. Therefore,

$$\Delta W_s = \frac{dW_s}{dHP} \Delta HP \qquad \text{(III-87)}$$

where:

$\frac{dW_s}{dHP}$ = the slope of the weight-power output curve at point $HP_s$.

Combining Eqs. (III-85), (III-86), and (III-87) gives:

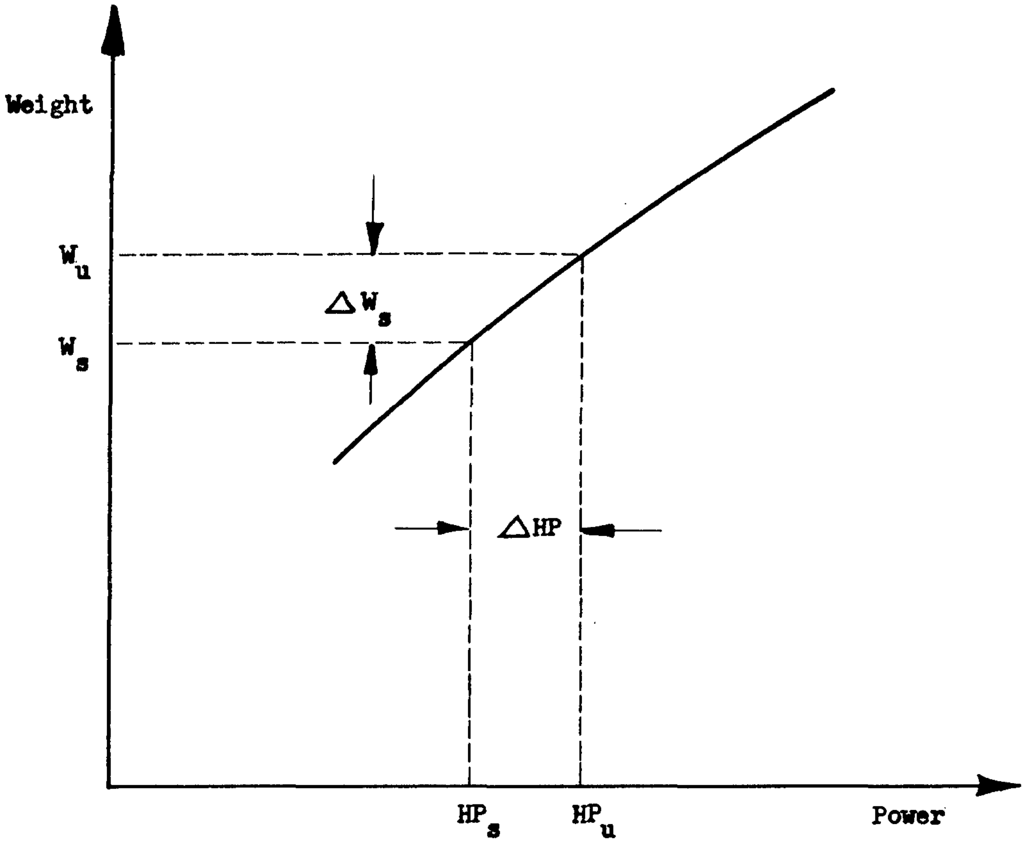
$$W_u = W_s + \frac{dW_s}{dHP}\ HP\ \frac{x^{*2}}{1 - x^{*2}} \qquad \text{(III-88)}$$

Eq. (III-88) expresses the weight of the pump and motor combination as the summation of a basic weight and an incremental weight. The basic weight corresponds to the weight of the units to deliver the prescribed power output in the absence of line losses. The incremental weight is due to the additional capacity the units must have to overcome the line losses. Fig. III-10 shows diagrammatically the method used in obtaining the weight of the pump and motor combination. In the evaluation of the weight of these units, care must be exercised to evaluate Eq. (III-87) at the proper power output level and correct pressure.

2. <u>Weight of Pump and Motor for a Constant Pressure, Variable Flow System</u>

The weight of a pump-motor combination for a constant pressure,

Fig. III-10 SCHEMATIC DIAGRAM SHOWING METHOD OF DETERMINING WEIGHT OF PUMP AND MOTOR REQUIRED FOR HYDRAULIC POWER TRANSMISSION SYSTEM




$HP_s$ = power output of system

$HP_u$ = power required from units

$\Delta HP$ = power losses due to line inefficiency

$W_s$ = weight of units required if no line losses were experienced

$W_u$ = weight of units required

$\Delta W_s$ = weight increment to pump and motor to provide $\Delta HP$ for line inefficiencies

variable flow system is derived in the same manner as that for a constant flow variable pressure system. However, the units must be evaluated to provide the maximum power output at rated pressure. Their displacement, consequently, will be larger than that of the constant flow variable pressure system units.

The weight of the pump and motor shown by Eq. (III-88) may be written as:

$$W_u = W_s + \frac{dWs}{dHP} HP \left[ \frac{1}{1 - x^{*2}} - 1 \right] \qquad \text{(III-89)}$$

Since the pump and motor must be large enough to supply maximum power, including losses, their weight should be evaluated at maximum power conditions. Eq. (III-89) then becomes:

$$W_u = W_s + \frac{dWs}{dHP_m} HP_m \left[ \frac{1}{1 - x_m^{*2}} - 1 \right] \qquad \text{(III-90)}$$

where:

$x_m^*$ = value of $x^*$ corresponding to $HP_m$

$HP_m$ = maximum power output of system

L. Weight of Oil Cooler

The weight of the oil cooler is proportional to the cooling area, which, according to Spec. AN-C-75, is proportional to the area of the heat exchanger face. For purposes of this study, a heat transfer rate of 15 BTU/min/in.$^2$ is assumed for the cooler, instead of the specified minimum of 14.4 BTU/min/in.$^2$.

A curve illustrating the weight variation of the cooler with change in cooling area per AN 4125 is shown in Fig. III-11.

The equation of this curve is:

$$W_c = 0.136 A_c + 2 \qquad \text{(III-91)}$$

The oil cooler must have sufficient face area to dissipate the heat losses of the pump and motor as well as those of the line. The losses of the pump and motor are evaluated at rated power. Cooling area is not provided for overload powers, since these are of short duration only.

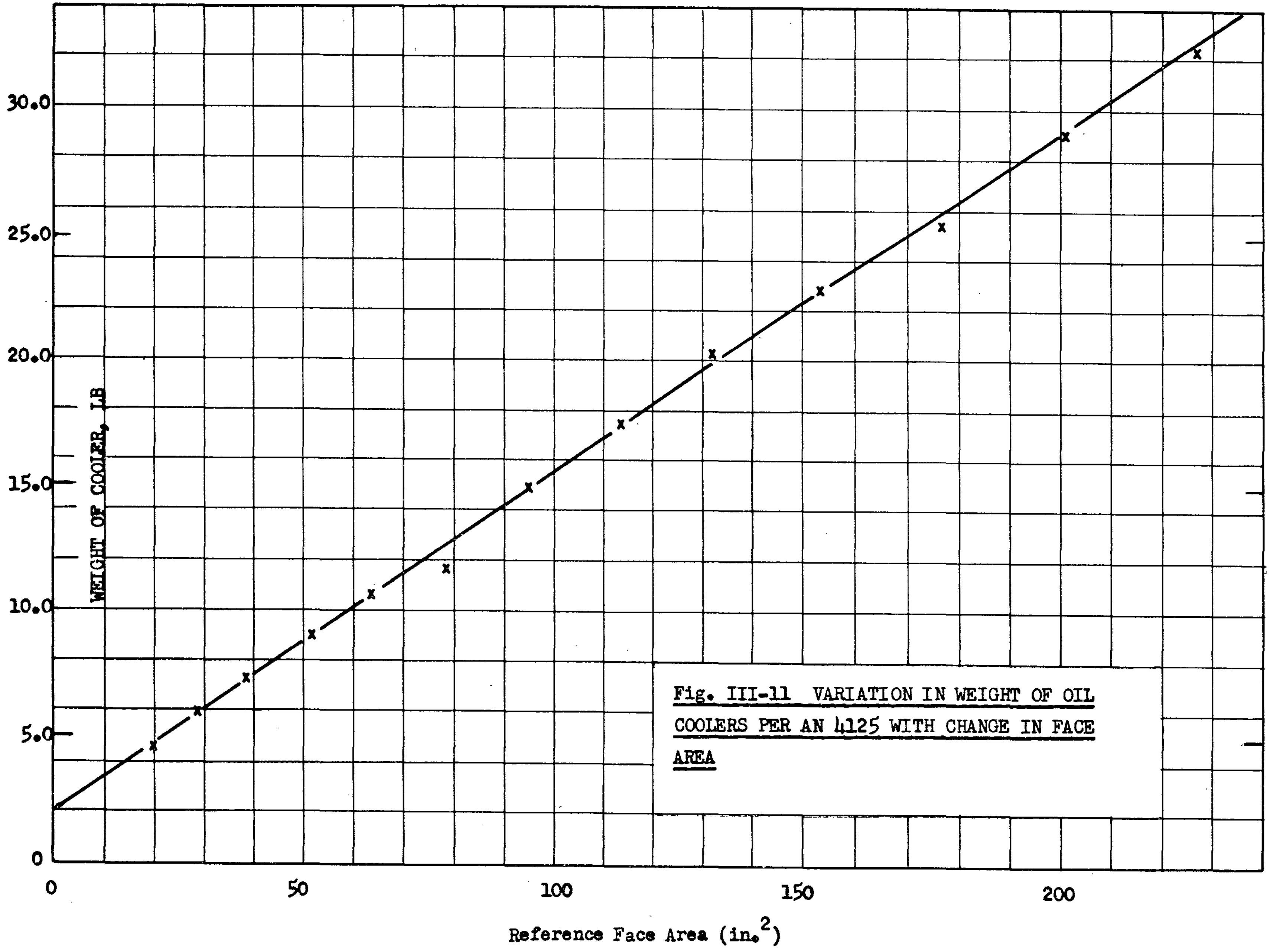



Fig. III-11 VARIATION IN WEIGHT OF OIL COOLERS PER AN 4125 WITH CHANGE IN FACE AREA

The power losses due to pump and motor at rated power are:

$$\Delta HP_u = \frac{HP_n}{\eta_b}\left[1 - \eta_b + \frac{1}{\eta_a \eta_L} - \frac{1}{\eta_L}\right] \qquad \text{(III-92)}$$

where:

$HP_n$ = rated power required by accessories

The line losses may be expressed as:

$$\Delta HP_L = \frac{HP_n}{\eta_b}\left(\frac{1}{\eta_L} - 1\right) \qquad \text{(III-93)}$$

Adding Eqs. (III-92) and (III-93) gives the total losses as:

$$\Sigma \Delta HP = \frac{HP_n}{\eta_b}\left[\frac{1}{\eta_a \eta_L} - \eta_b\right] \qquad \text{(III-94)}$$

1. <u>Weight of Oil Cooler for a Constant Flow, Variable Pressure System</u>

Eq. (III-94) may be written in the form:

$$\Sigma \Delta HP = \frac{HP_n}{\eta_a \eta_b}\left(\frac{1}{\eta_L} - \eta_a \eta_b - 1 + 1\right) \qquad \text{(III-95)}$$

Regrouping terms, substituting the value of $\eta_L$ as shown in Eq. (III-19) and evaluating the resulting $x^*$ expression at $HP_{cr}$ gives:

$$\Sigma \Delta HP = \frac{HP_n}{\eta_a \eta_b}\left(1 - \eta_a \eta_b\right) + \frac{HP_{cr}}{\eta_a \eta_b}\left(\frac{x^{*2}}{1 - x^{*2}}\right) \qquad \text{(III-96)}$$

Converting the power losses to heat energy, and dividing by 15 BTU/min/in.$^2$, gives the required face area of the cooler as

$$A_c = 2.83\,\frac{HP_n}{\eta_a \eta_b}\left(1 - \eta_a \eta_b\right) + 2.83\,\frac{HP_{cr}}{\eta_a \eta_b}\left(\frac{x^{*2}}{1 - x^{*2}}\right) \qquad \text{(III-97)}$$

Combining Eqs. (III-91) and (III-97) gives, as the weight of the oil cooler,

$$W_c = \frac{0.385}{\eta_a \eta_b}\left[ HP_n (1 - \eta_a \eta_b) + HP_{cr} \frac{x^{*2}}{1 - x^{*2}} \right] + 2 \qquad \text{(III-98)}$$

2. <u>Weight of Oil Cooler for a Constant Pressure, Variable Flow System</u>

For a constant pressure, variable flow system, the required cooling capacity is considered to be one-half of the energy losses at maximum power. This assumption provides for a somewhat greater cooling capacity than is required at the normal rated power of the system. Because of the relatively light weight of the cooler, however, it is felt that this assumption is justified.

It should be noted that if the system is to be utilized in a manner which may require operation at maximum power output for any except extremely short periods of time, adequate cooling capacity must be provided. Under these circumstances, the losses should be evaluated as the total losses at maximum power output.

The total energy losses as shown by Eq. (III-94) are:

$$\Sigma \Delta HP = \frac{HP}{\eta_b}\left[ \left( \frac{1}{\eta_a \eta_L} \right) - \eta_b \right] \qquad \text{(III-99)}$$

where:

$\Sigma \Delta HP$ = total losses

Since the losses are to be assumed as one-half of the maximum, Eq. (III-99) becomes:

$$\Sigma \Delta HP = \frac{HP_m}{2\eta_b}\left( \frac{1}{\eta_a \eta_L} - \eta_b \right) = \frac{HP_m}{2}\left[ \frac{1}{\eta_a \eta_b} \left( \frac{1}{1 - x_m^{*2}} \right) - 1 \right] \qquad \text{(III-100)}$$

Multiplying Eq. (III-100) by the thermal equivalent of power (42.42 BTU/HP/min), dividing by 15 BTU/min/in.$^2$, to determine the face area and combining the results with Eq. (III-91) gives the weight of the oil cooler as:

$$W_c = 0.1925\ HP_m \left[ \frac{1}{\eta_a \eta_b} \left( \frac{1}{1 - x^{*2}} \right) - 1 \right] + 2 \qquad \text{(III-101)}$$

M. Total Weight of an Optimum Constant Flow, Variable Pressure System

1. Total Weight of a Constant Flow, Variable Pressure System

The total weight of a hydraulic power transmission system is the sum of the weights of its components. Thus:

$$\Sigma W = W_u + W_c + W_L + W_r + W_F + W_{cL} + W_k \tag{III-102}$$

where:

$\Sigma W$ = total weight of system

$W_u$ = weight of pump and motor (Eq. III-88)

$W_c$ = weight of oil cooler (Eq. III-98)

$W_L$ = weight of lines (Eq. III-53)

$W_r$ = weight of reservoir (Eq. III-59)

$W_F$ = weight of fuel (Eq. III-71)

$W_{cL}$ = weight of control lines

$W_k$ = weight of any additional airframe structure due to the system installation, and weight of fuel required to overcome any aerodynamic drag chargeable to the system

Combining the indicated equations, regrouping terms, and substituting the value of $\Omega$ and $G$ in Eqs. (III-22) and (III-7) respectively, the total weight of the system may be written

$$\begin{aligned}\Sigma W = {} & W_s + 0.385\,\frac{HP_n}{\eta_a \eta_b}\left[1 - \eta_a \eta_b\right] + 2 + \frac{C''_{PX}\,\tau\,HP_{cr}}{\eta_a \eta_b} + JL_3 + W_k \\ & + \left[\frac{dW_s}{dHP}\,HP_{cr} + 0.385\,\frac{HP_{cr}}{\eta_a \eta_b} + \frac{C''_{PX}\,\tau\,HP_{cr}}{\eta_a \eta_b}\right]\frac{x^{*2}}{1 - x^{*2}} \\ & + \left[\frac{\pi}{4}(\alpha + \beta\tau)\,\gamma_f^{\,h} + \phi\right]\frac{\left[8\,\gamma_f\,\Sigma K\right]^{1/2}\,550\,L\,HP_{cr}}{\pi\,g^{1/2}\left(P_{cr}\right)^{3/2}\eta_b\,x^{*}\,(1 - x^{*2})}\end{aligned} \tag{III-103}$$

2. Relationship Between $x^*$ and $x^*_m$

Eq. (III-103) expresses the weight of the system in terms of $x^{*2}$, or $\Delta P/P_{cr}$, for which the optimum value is required. Since the value of $P_{cr}$ is not known, it must be expressed in terms of known values. The value of $P_{cr}$ can be expressed in terms of the known maximum pressure $P_m$ by the use of Eq. (III-37). This gives:

$$\frac{\Delta P}{P_{cr}} = \frac{\Delta P}{\Delta P + \dfrac{P_m - \Delta P}{HP_m} HP_{cr}} = \frac{\dfrac{\Delta P}{P_m}}{\dfrac{\Delta P}{P_m}\left(1 - \dfrac{\Delta P}{P_m}\right)\dfrac{HP_{cr}}{HP_m}} \qquad \text{(III-104)}$$

Let,

$$x_m^{*2} = \frac{\Delta P}{P_m} \qquad \text{(III-105)}$$

then

$$\frac{x^{*2}}{1 - x^{*2}} = \frac{x_m^{*2}\ HP_m}{(1 - x_m^{*2})\ HP_{cr}} \qquad \text{(III-106)}$$

and

$$\frac{1}{x^*(1 - x^{*2})} = \frac{HP_m}{HP_{cr}}\left(\frac{P_{cr}}{P_m}\right)^{3/2}\frac{1}{x_m^*(1 - x_m^{*2})} \qquad \text{(III-107)}$$

Eq. (III-103) then may be rewritten:

$$\Sigma W = W_s + 0.385\frac{HP_n}{\eta_a\eta_b}(1 - \eta_a\eta_b) + 2 + \left(\frac{C''_{PX}\ HP_{cr}\ \tau}{\eta_a\ \eta_b}\right) + JL_3 + W_k$$

$$+ \left[\frac{dW_s}{dHP_m} + \frac{0.385}{\eta_a\ \eta_b} + \frac{C''_{PX}\ \tau}{\eta_a\ \eta_b}\right] HP_m \frac{x_m^{*2}}{1 - x_m^{*2}}$$

$$+ \left[\frac{\pi}{4}(\alpha + \beta\tau)\gamma_f h + \Phi\right]\frac{\left[8\gamma_f \Sigma K\right]^{1/2}}{\pi g^{1/2}\ \eta_b}\ \frac{550}{P_m^{3/2}}\ \frac{L\ HP_m}{x_m^{*2}(1 - x_m^{*2})} \qquad \text{(III-108)}$$

3. <u>Optimum Line Pressure Drop for Constant Flow, Variable Pressure System</u>

Eq. (III-108) expresses the total weight of a constant flow, variable pressure hydraulic transmission system designed to deliver a given maximum power, in terms of the variable, $x_m^{*2}$, which is defined as $\Delta P/P_m$. It should be noted that the weight of the system is dependent upon the maximum power output and the duration of system operation. The only other power rating, $HP_n$, appears in a constant term. The line losses are constant regardless of the power output, since the flow is constant.

The optimum value of $\Delta P/P_m$ is obtained by differentiating $\Sigma W$, as expressed by Eq. (III-108) with respect to $x_m^*$, and equating the result to zero. This results in:

$$\frac{2\left(\dfrac{dW_s}{dHP_m} + \dfrac{0.385}{\eta_a \eta_b} + \dfrac{C''_{PX}\,\tau}{\eta_a \eta_b}\right)}{\left[\dfrac{\pi}{4}(\alpha + \beta\tau)\gamma_f h + \Phi\right](L)\,\dfrac{\left[8\,\gamma_f\,\Sigma K\right]^{1/2} 550}{\pi\, g^{1/2}\,\eta_a\, P_b^{3/2}}} = \frac{1 - 3x_m^{*2}}{x_m^{*3}} \qquad \text{(III-109)}$$

Let,

$$A = W_s + \frac{0.385}{\eta_a \eta_b}\left(1 - \eta_a \eta_b\right) + 2 + \frac{C''_{PX}\,\tau_{HP}}{\eta_a \eta_b} + JL_3 + W_k \qquad \text{(III-110)}$$

$$B = \frac{dW_s}{dHP_m} + \frac{0.385}{\eta_a \eta_b} + \frac{C''_{PX}\,\tau}{\eta_a \eta_b} \qquad \text{(III-111)}$$

$$C = 550\left[\frac{\pi}{4}(\alpha + \beta\tau)\gamma_f h + \Phi\right](L)\,\frac{\left[8\,\gamma_f\,\Sigma K\right]^{1/2}}{\pi\, g^{1/2}\,\eta_b\, P_m^{3/2}} \qquad \text{(III-112)}$$

Eq. (III-109) may be written:

$$\frac{1 - 3\,x_m^{*2}}{x_m^{*3}} = \frac{2B}{C} \qquad \text{(III-113)}$$

Fig. III-12 is a graphical representation of this relationship. After evaluating the ratio 2B/C, the optimum value of $x_m^*$ can be obtained from this curve. Solution of Eq. (III-26) using this value of $x_m^*$ produces the optimum line diameter.

### 4. Total Weight of an Optimum Constant Flow Variable Pressure System

The total weight of the optimum system is found by substitution of the optimum $x^*$ into Eq. (III-108). This may be simplified by use of Eqs. (III-110), (III-111) and (III-112) to read

$$\Sigma W = A + B\, HP_m \frac{x_m^{*2}}{1 - x_m^{*2}} + C \frac{HP_m}{x_m^* \left( 1 - x_m^{*2} \right)} \tag{III-114}$$

## N. Weight of an Optimum Constant Pressure, Variable Flow System

### 1. Total Weight of a Constant Pressure, Variable Flow System

Substituting Eqs. (III-90), (III-101), (III-53), (III-59) and (III-71) into Eq. (III-101), the weight of the system may be expressed as:

$$\begin{aligned} W = {} & W_{sm} + \frac{dW_s}{dHP_m} HP_m \left[ \frac{1}{1 - x_m^{*2}} - 1 \right] - 0.1925\, HP_m \\ & + \frac{0.1925\, HP_m}{\eta_a \; \eta_b} \; \frac{1}{1 - x_m^{*2}} + \frac{C''_{PX} \, \tau \, HP_{cr}}{\eta_a \; \eta_b \, (1 - x^{*2})} \\ & + \frac{550 \, \Phi \, L \left[ 8 \, \gamma_f \, \Sigma K \right]^{1/2} HP_{cr}}{\pi \, g^{1/2} \, P^{3/2} \, \eta_b x^* \, (1 - x^{*2})} \\ & + \frac{550 \, (\alpha + \beta \tau) \, L \, \gamma_f \, h \left[ 8 \, \gamma_f \, \Sigma K \right]^{1/2} HP_{cr}}{4 \, g^{1/2} \, P^{3/2} \, \eta_b \, x^* \, (1 - x^{*2})} \\ & + JL_3 + W_k \end{aligned} \tag{III-115}$$

[redacted]

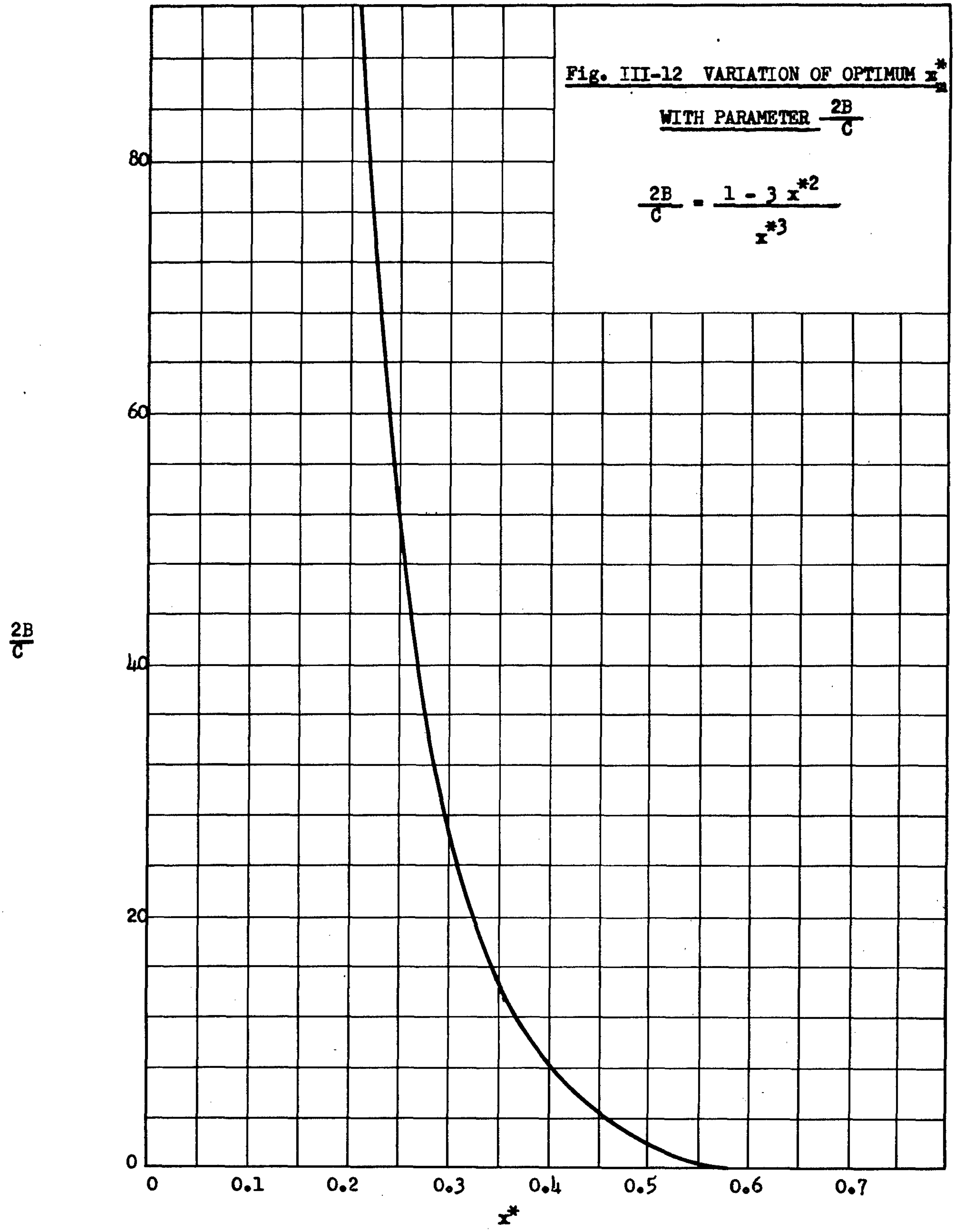



Fig. III-12 VARIATION OF OPTIMUM $x_m^*$ WITH PARAMETER $\frac{2B}{C}$



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Rearranging terms, Eq. (III-115) becomes:

$$\Sigma W = W_{sm} - \frac{dW_s}{dHP_m} HP_m - 0.1925 \; HP_m + 2 + JL_3 + W_k$$

$$+ \frac{dW_s}{dHP_m} HP_m \frac{1}{1 - x_m^{*2}} + \frac{0.1925}{\eta_a \eta_b} \frac{HP_m}{1 - x_m^{*2}}$$

$$+ \frac{C''_{PX} \, \tau \, HP_{cr}}{\eta_a \; \eta_b} \frac{1}{1 - x^{*2}}$$

$$+ \frac{550 \; L \left[ \Phi + \frac{\pi}{4} \gamma_f \, h \; (\alpha + \beta \tau) \right] \left[ 8 \, \gamma_f \, \Sigma K \right]^{1/2} HP_{cr}}{\pi \, g^{1/2} \; p^{3/2} \; \eta_b \, x^* \, (1 - x^{*2})} \tag{III-116}$$

Let:

$$H = W_s - \frac{dW_s}{dHP_m} HP_m - 0.1925 \; HP_m + 2 + JL_3 + W_k \tag{III-117}$$

$$R = \frac{dW_s}{dHP_m} + \frac{0.1925}{\eta_a \eta_b} \tag{III-118}$$

$$M = \frac{C''_{PX} \, \tau}{\eta_a \; \eta_b} \tag{III-119}$$

$$S = \frac{550 \; L \left[ \Phi + \frac{\pi}{4} \gamma_f \, h \; (\alpha + \beta \tau) \right] \left[ 8 \; \gamma_f \, \Sigma K \right]^{1/2}}{\pi \, g^{1/2} \; p^{3/2} \; \eta_b} \tag{III-120}$$

Then, Eq. (III-116) may be written as:

$$\Sigma W = H + R \frac{HP_m}{1 - x_m^{*2}} + M \frac{HP_{cr}}{1 - x^{*2}} + S \frac{HP_{cr}}{x^* \, (1 - x^{*2})} \tag{III-121}$$

The minimum system weight will occur at some value of $x^*$. This optimum value of $x^*$ may be found by differentiating $\Sigma W$ with respect to $x^*$. However, it is first necessary to find the relationship between $x_m^*$ and $x^*$ and to express $x_m^*$ in terms of $x^*$.

2. Relationship Between $x_m^*$ and $x^*$

The basic relationship between power output and $x^*$ is shown by Eq. (III-40).

$$\frac{550 \quad G^{1/2} \quad HP}{P \quad \eta_b \quad D^2} = x^* (1 - x^{*2}) \qquad \text{(III-122)}$$

Substituting the value of $G = \dfrac{\gamma_f \quad \Sigma K}{\pi^2 \quad g \quad P}$, as shown by Eq. (III-7).

$$\frac{550 \quad \left[8 \quad \gamma_f \quad \Sigma K\right]^{1/2} \quad HP}{\eta_b \quad \pi g^{1/2} \quad P^{3/2} \quad D^2} = x^* - x^{*3} \qquad \text{(III-123)}$$

The practical working range of Eq. (III-123) is from $x^* = 0$ to $x^* = \dfrac{1}{\sqrt{3}} = 0.57735$. Approximating Eq. (III-123) within the working range with a quadratic equation by a modified method of least squares, produces the approximation:

$$\frac{550 \left[8 \quad \gamma_f \Sigma K\right]^{1/2} \quad HP}{\eta_b \quad \pi \, g^{1/2} \quad P^{3/2} \quad D^2} = -0.96225 \quad x^{*2} + 1.2444 \, x^* - 0.01283$$

(III-124)

Fig. III-13 shows graphically the approximation expressed by Eq. (III-124) compared to the original equation shown in Eq. (III-123). Within the working range of $x^* = 0.06$ to $x^* = 0.57735$, the maximum error is 3.85 per cent at $x^* = 0.14$.

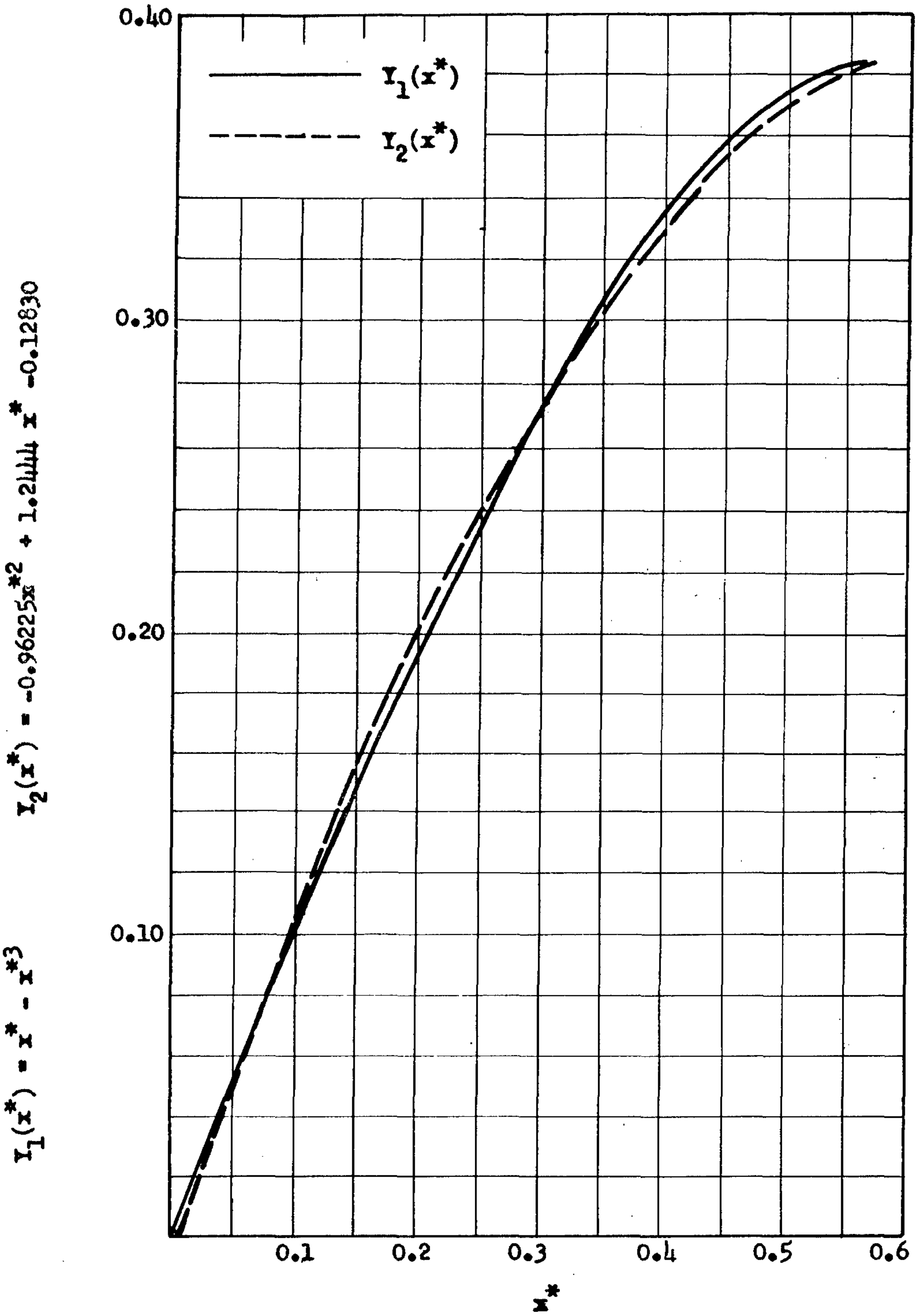


Fig. III-13 COMPARISON BETWEEN ORIGINAL FUNCTION $Y_1(x^*)$ AND APPROXIMATE FUNCTION $Y_2\ (x^*)$

Eq. (III-124) shows the relationship between cruise power output $HP_{cr}$ and the corresponding value of $x^*$.

The maximum power output may be expressed as:

$$\frac{550 \quad 8 \; \gamma_f \, \Sigma K^{1/2}}{\eta_b \, \pi_g^{1/2} \; P^{3/2} \; \Gamma^2} HP_m = -0.96225 \, x_m^{*2} + 1.2444 \, x_m^* - 0.01283 \tag{III-125}$$

where

$x_m^*$ = value of $x^*$ at maximum power output conditions

Dividing Eq. (III-125) by Eq. (III-124) gives:

$$\frac{HP_m}{HP_{cr}} = \frac{-0.96225 \, x_m^{*2} + 1.2444 \, x_m^{*2} - 0.01283}{-0.96225 \, x^{*2} + 1.2444 \, x^* - 0.01283} \tag{III-126}$$

Solving for $x_m^*$ produces:

$$x_m^* = \frac{-1.2444}{-2(0.96225)} - \sqrt{\frac{(1.2444)^2}{4(-0.96225)^2} - \frac{1}{-0.96225}\left[(-0.01283) - (-0.96225 \, x^{*2} + 1.2444 \, x^* - 0.01283)\frac{HP_m}{HP_{cr}}\right]} \tag{III-127}$$

This may be simplified to

$$x_m^* = 0.6466 - \sqrt{0.39454 + 1.0392 \, (0.96225 \, x^{*2} - 1.2444 \, x^* + 0.01283) \frac{HP_m}{HP_{cr}}} \tag{III-128}$$

Let:

$$z = \sqrt{0.39454 + 1.0392\,(0.96225\,x^{*2} - 1.2444\,x^{*} + 0.01283)\,\frac{HP_m}{HP_{cr}}} \qquad \text{(III-129)}$$

Then:

$$x^{*}_{m} = 0.6466 - z \qquad \text{(III-130)}$$

Fig. III-14 shows the relationship between $x^{*}_{m}$ and $x^{*}$ as expressed by Eq. (III-128) for $\frac{HP_m}{HP_{cr}}$ ratios of 1,2,3, and 4. Values of $x^{*}_{m}$ for any $x^{*}$ and intermediate values of $\frac{HP_m}{HP_{cr}}$ may be obtained by plotting the $x^{*}_{m}$ values for the two adjacent ratios of $\frac{HP_m}{HP_{cr}}$ and interpolating therefrom.

3. <u>Determination of the Optimum Line Pressure Drop For a Constant Pressure, Variable Flow System</u>

Substituting the value of $x^{*}_{m}$ into $(1 - x^{*2}_{m})$ shown in Eq. (III-119) gives

$$\frac{1}{1 - x^{*2}_{m}} = \frac{1}{1 - 0.41813 + 1.29332\,z - z^{2}} \qquad \text{(III-131)}$$

Eq. (III-121) may then be written as:

$$\Sigma W = H + \frac{R\,HP_{cr}}{0.58187 + 1.2932z - z^{2}} + \frac{M\,HP_{cr}}{1 - x^{*2}} + \frac{S\,HP_{cr}}{x^{*}\,(1 - x^{*2})} \qquad \text{(III-132)}$$

Differentiating Eq. (III-132) with respect to $x^{*}$, and equating to zero, the following expression is obtained:

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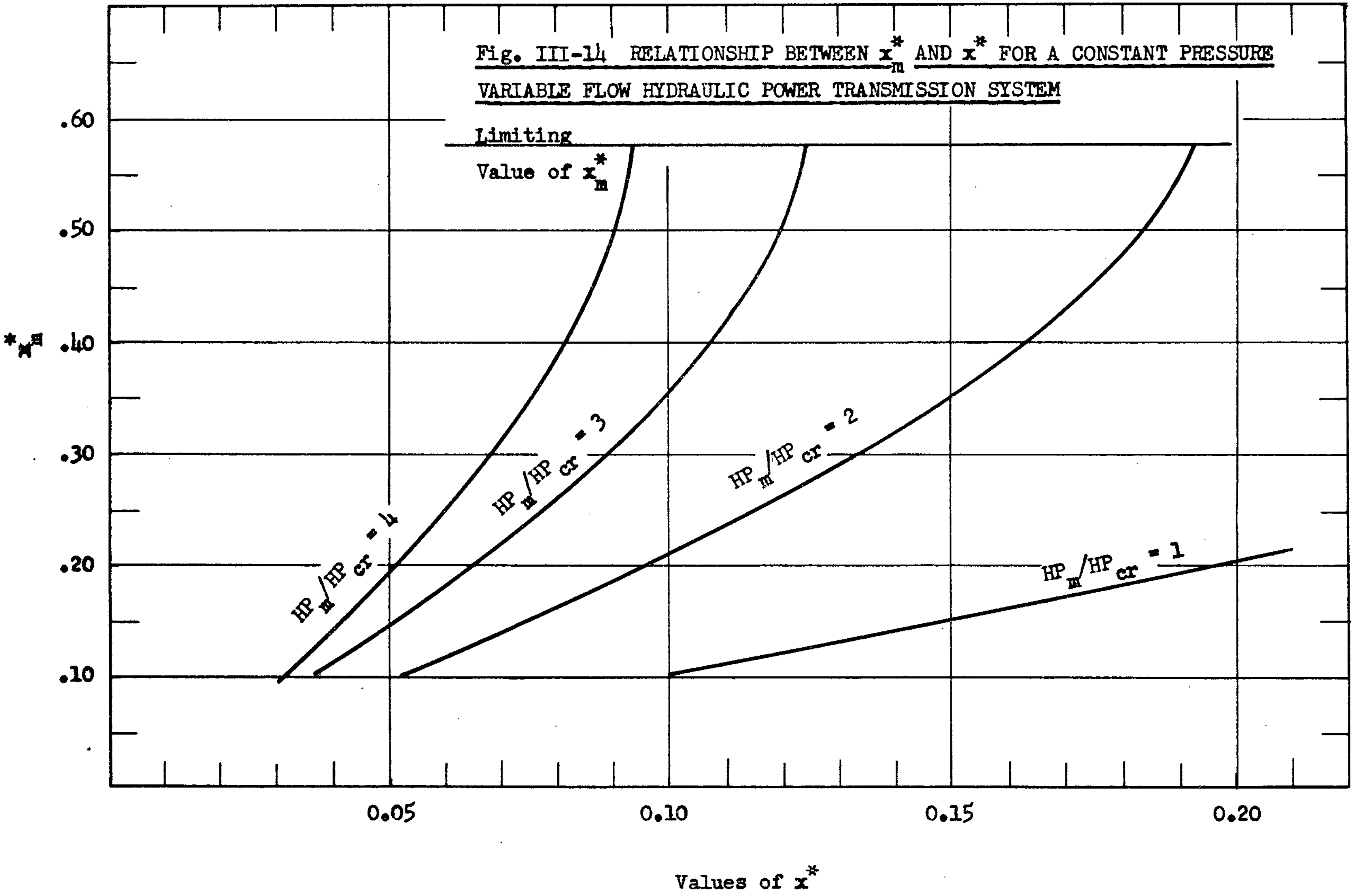
Fig. III-14 RELATIONSHIP BETWEEN $x_m^*$ AND $x^*$ FOR A CONSTANT PRESSURE VARIABLE FLOW HYDRAULIC POWER TRANSMISSION SYSTEM






[redacted]

$$\frac{d\,\Sigma W}{dx^*} = \frac{R\,(0.6466 - z)^3\,(2\,x^* + 1.2932)}{x^{*2}\,(1 - x^{*2})^2\,z}\,HP_{cr} + \frac{2\,M\,x^*\,HP_{cr}}{(1 - x^{*2})^2}$$

$$+ \frac{3\,x^{*2} - 1}{x^{*2}\,(1 - x^{*2})^2}\,S\,HP_{cr} = 0 \qquad \text{(III-133)}$$

Dividing by the second term, and transposing the third term,

$$\frac{(0.6466 - z)^3\,(2\,x^* + 1.2932)}{z\,x^{*3}}\,\frac{R}{M} + 2 = \frac{S}{M}\,\frac{1 - 3\,x^{*2}}{x^{*3}} \qquad \text{(III-134)}$$

Eq. (III-134) expresses the change in the slope of the total weight curve of the system with change in the independent variable, $x^*$, and the parameters, R/M and S/M. By calculating the values of the left member of the equation for various values of $x^*$, keeping R/M constant, a curve of the left member of the equation can be drawn. Assuming successively different values for the parameter R/M, a family of curves is obtained. The same procedure can be followed with the right member of the equation. The resulting curves are shown in Part 1 of this report (Ref. III-4). Fig. III-7 of Part 1 is a plot of the curves obtained from Eq. (III-134) for the ratio $\frac{HP_m}{HP_{cr}} = 1$. Figs. III-8, III-9, and III-10 of Part 1 are similar curves calculated for $\frac{HP_m}{HP_{cr}}$ ratios of 2, 3, and 4 respectively.

The optimum value of $x^*$ is obtained by locating the intersection of the proper curves of the left and the right members of the equation, which for convenience are called the R/M curve and the S/M curve, respectively. The optimum value of $x^*$ lies directly below the intersection of the above two curves.

4. Weight of an Optimum Constant Pressure Variable Flow System

The total weight of the system is obtained by substitution of the optimum values of $x^*$ and $x^*_m$ (obtained from Fig. III-14) into Eq. (III-121). The weights of the components of the system are similarly obtained, using the appropriate equation derived in this report.

## REFERENCES

III-1 Nels M. Sverdrup, Calculating the Energy Losses in Hydraulic Systems, Product Engineering, pp 146 - 152, April 1951.

III-2 James E. Campbell, Investigation of the Fundamental Characteristics of High Performance Hydraulic Systems, USAF Technical Report No. 5997, June 1950.

III-3 Military Specification MIL-E-5008, Engines, Aircraft, Turbojet, Model Specification for, 27 July 1951.

III-4 **Evaluation of Aircraft Accessory Power Transmission Systems by Selected Analytical Methods, Part 1**

III-5 Army-Navy Aeronautical Specification AN-P-11b, Pumps, Power Driven Hydraulic, 17 June, 1948.

III-6 Military Specification MIL-C-5637, Coolers, Oil, Tubular, Aircraft, 15 February, 1950.

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WEIGHT ANALYSIS OF THE ELECTRIC

POWER TRANSMISSION SYSTEM

Section IV

## A. Introduction

The material presented in this section consists of derivations of equations used in the analysis of the electric power transmission system. In the case where the result has been shown graphically in Part 1 of this report, the respective figure is repeated here for easier reference.

## B. Nomenclature

| | |
|---|---|
| $A$ | area, $ft^2$ |
| $E$ | voltage, volt |
| $I$ | current, amp |
| $L$ | length, ft |
| $W$ | weight, lb |
| $n$ | number |
| pf | power factor |
| $\gamma$ | weight density, $lb/ft^3$ |
| $\Delta$ | difference |
| $\eta$ | efficiency, per cent |
| $\rho$ | resistivity, ohm-ft |

Subscripts

| | |
|---|---|
| c | conductor or cable |
| g | generator or alternator |
| i | insulation |
| j | single |
| m | motor |



[redacted]

o  output

r  rated

C. Determination of Combined Cable Density

Since the weight of the cable is composed of that of the insulation and the conductor, the total weight may be expressed by

$$W_c = \gamma_c AL + \gamma_i A_i L \qquad \text{(IV-1)}$$

where

$W_c$ = weight of cable, lb

$\gamma_c$ = weight density of the conductor, lb/ft$^3$

A = cross-sectional area of the conductor, ft$^2$

$\gamma_i$ = weight density of the insulation, lb/ft$^3$

$A_i$ = cross-sectional area of the insulation, ft$^2$

L = length of cable, ft

By grouping terms, Eq. (IV-1) can be written as

$$W_c = \left( \gamma_c + \gamma_i \frac{A_i}{A} \right) AL = \gamma AL \qquad \text{(IV-2)}$$

where

$\gamma$ = combined weight density of cable and insulation, lb/ft$^3$

The value of this combined density can be closely approximated for the larger size cables if a plot of the weight of the cable per unit length is made against conductor area. Then since those points which correspond to the larger cables lie in a straight line, the slope of the line gives a numerical evaluation of $\gamma$. Fig. IV-1 shows the determination of $\gamma$ for cable using a copper conductor.

D. Weight of the Cables

The weight of the cables is given by

$$W_c = \gamma A L n_c = \frac{\gamma L I}{\frac{I}{A}} n_c \qquad \text{(IV-3)}$$

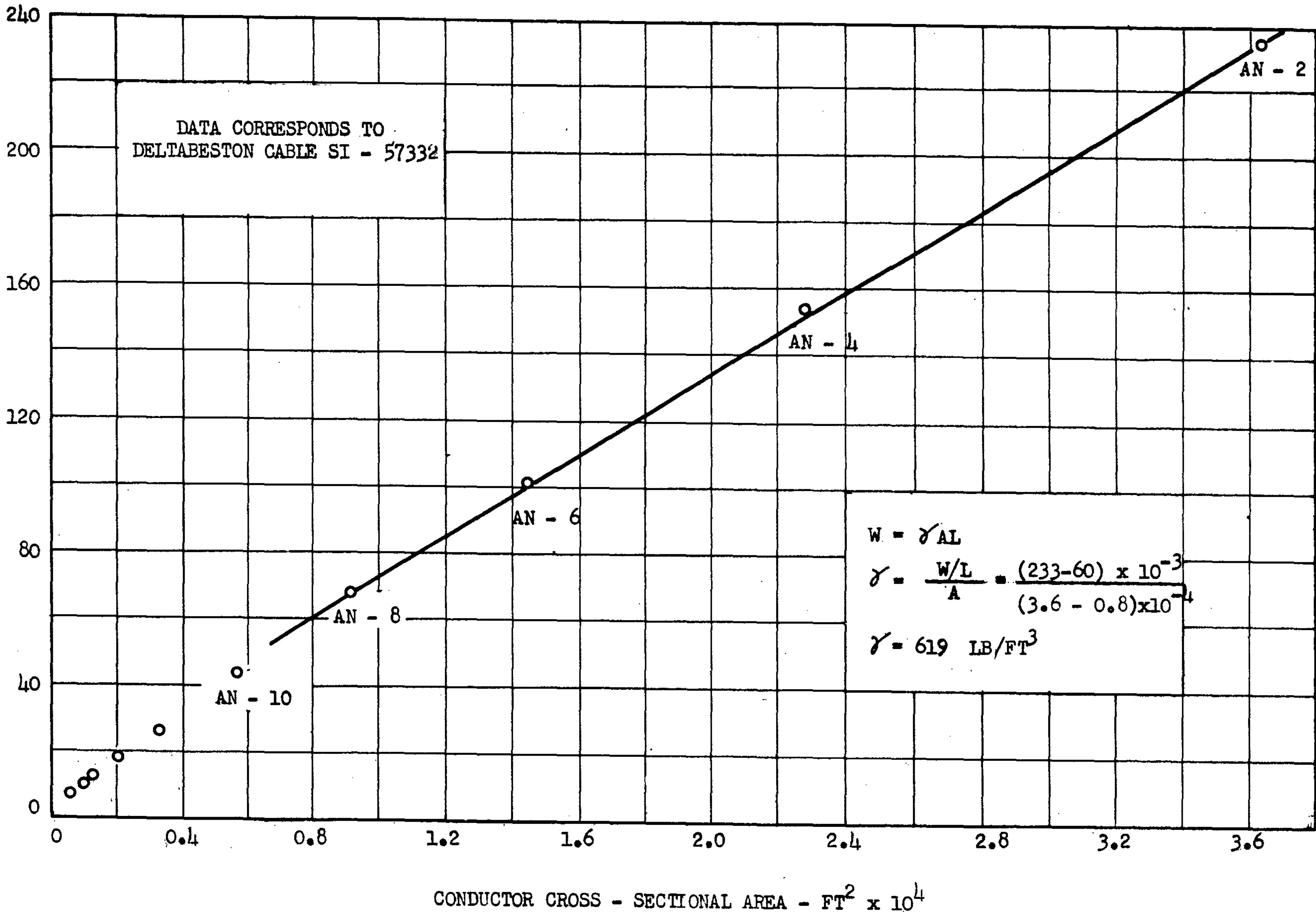



Fig. IV-1 DETERMINATION OF COMBINED COPPER DENSITY

where

$\gamma$ = combined weight density of cable and insulation, lb/ft$^3$

$A$ = cross-sectional area of conductor, ft$^2$

$L$ = length of cable, ft

$n_c$ = number of cables in system (one or two-wire system)

$I$ = current carried by cable, amp

The voltage drop along a single cable of length $L$ is given by

$$\Delta E_j = \frac{I}{A} L \rho \qquad \text{(IV-4)}$$

where

$\Delta E_j$ = voltage drop, volt

$\rho$ = resistivity of the conductor, ohm-ft

The total voltage drop between the generator and the motor, $\Delta E$, is equal to $\Delta E_i$ for a grounded system. For a two-wire system the total voltage drop $\Delta E$ is equal to $2 \Delta E_i$ since an additional voltage drop occurs in the return line. The current density can now be expressed in terms of the total voltage drop.

For one-phase systems

$$\frac{I}{A} = \frac{\Delta E}{n_c L \rho} \qquad \text{(IV-5)}$$

and for a grounded three-phase system

$$\frac{I}{A} = \frac{\Delta E}{L \rho} \qquad \text{(IV-6)}$$

The current carried by the cable for a given power output can be obtained from Table **IV-1 for the several systems analyzed.**

If the equations for the current and current density are introduced into Eq. (IV-3), the following expressions are obtained for the total weight of the transmission cables:

for d-c systems:

$$W_c = \frac{\gamma \rho n_c^2 \quad L^2 \left(P_{or}/\eta_m\right)}{E_g \left[1 - \frac{\Delta E}{E_g}\right] \Delta E} \tag{IV-7}$$

for one-phase a-c systems:

$$W_c = \frac{\gamma \rho n_c^2 \quad L^2 \left(P_{or}/\eta_m\right)}{E_g \,(pf) \left[1 - \frac{\Delta E}{E_g (pf)}\right] \Delta E} \tag{IV-8}$$

and for three-phase neutral grounded systems:

$$W_c = \frac{\gamma \rho L^2 \left(P_{or}/\eta_m\right)}{E_g (pf) \left[1 - \frac{\Delta E}{E_g (pf)}\right] \Delta E} \tag{IV-9}$$

# TABLE IV-1

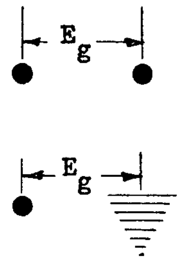
## Current Carried by Cable for a Given Power Output

### D-C Systems

2-wire system

or

grounded return (1 wire)

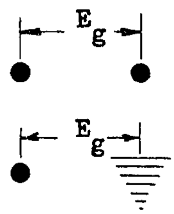



$$I = \frac{P_{or}/\eta_m}{E_g\left(1 - \frac{\Delta E}{E_g}\right)}$$

### A-C Systems

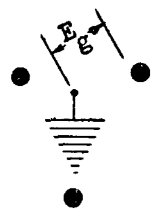
1-phase, 2-wire system

1-phase, grounded return




$$I = \frac{P_{or}/\eta_m}{E_g\,(pf)\left[1 - \frac{\Delta E}{E_g\,(pf)}\right]}$$

3-phase, neutral grounded




$$I = \frac{P_{or}/\eta_m}{3E_g\,(pf)\left[1 - \frac{\Delta E}{E_g\,(pf)}\right]}$$

$P_{or}$ = rated power output of motor, watt

$E_g$ = generator or alternator voltage, volt

$\Delta E$ = rated voltage drop between generator and motor, volt

$\eta_m$ = motor efficiency

pf = power factor

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# WEIGHT ANALYSIS OF A MECHANICAL POWER TRANSMISSION SYSTEM

## Section V

### A. Introduction

The derivation and source of the following factors used in the evaluation of the mechanical system may not be apparent:

Torque parameter, $\mathcal{T}$, and the solid shaft diameter, $D_s$

Critical speed parameter, $L\sqrt{N_{cr}}$

Weight of the pillow block, $W_p$

Weight of the shaft per unit length, $\frac{W}{L}$

Weight of fuel, $W_F$

Weight of shaft housing, $W_h$

Optimum shaft diameter, $(D_o)_{opt}$

The equations and curves for these factors are derived and explained in this section.

### B. Nomenclature

| | |
|---|---|
| $A$ | cross-sectional area which must be added to standard pillow block to provide clearance for torque tube, in.$^2$ |
| $c$ | radial clearance between outside diameter of the shaft and the inside diameter of the shaft housing, in. |
| $C_1$ | parameter defined by Eq. (V-59) |
| $C_2$ | parameter defined by Eq. (V-60) |
| $C_{PX}$ | thrust correction factor due to power extraction |
| $C'_{PX}$ | fuel flow correction factor due to power extraction |
| $C''_{PX}$ | specific fuel consumption of transmission system, lb/HP-hr |
| $D_b$ | outside diameter of bearing, in. |
| $D_h$ | inside diameter of housing, in. |



[redacted]

| | |
|---|---|
| $D_i$ | inside diameter of torque tube, in. |
| $D_o$ | outside diameter of torque tube, in. |
| $D_s$ | solid shaft diameter, in. |
| $E$ | modulus of elasticity, lb/in.$^2$ |
| $F_n$ | thrust of unburdened engine, lb |
| $g$ | acceleration due to gravity, in./sec$^2$ |
| $HP_r$ | normal rated horsepower output of transmission system |
| $HP_c$ | horsepower output of transmission system at cruise conditions |
| $HP_{REF}$ | reference horsepower of engine |
| $HP_x$ | horsepower extracted from engine |
| $I$ | rectangular moment of inertia, in.$^4$ |
| $K$ | diameter ratio, $D_i/D_o$ |
| $K_t$ | shock factor |
| $\mathcal{L}$ | total distance accessory power to be transmitted, ft |
| $L$ | shaft unit length, in. |
| $L_{cu}$ | length of coupling unit, in. |
| $L_s$ | length of solid shaft, in. |
| $n$ | number of shaft units |
| $N$ | normal rated shaft speed, rpm |
| $N_{cr}$ | critical speed of shaft, rpm |
| $R$ | area ratio of pillow block |
| $s_s$ | design shear stress, psi |
| $T$ | torque, lb-in. |
| $t$ | practical minimum wall thickness of torque tube, in. |
| $t_h$ | wall thickness of shaft housing, in. |
| $T_s$ | static torque, lb-in. |

| | |
|---|---|
| $W$ | torque tube weight, lb |
| $w_{ad}$ | width of adapter, in. |
| $w_p$ | width of the pillow block, in. |
| $W_b$ | bearing weight, lb |
| $\Sigma W$ | total weight of transmission system, lb |
| $W_{cs}$ | weight of the constant speed drive, lb |
| $W_{cu}$ | coupling unit weight, lb |
| $W_F$ | weight of fuel required to operate the accessory system for a given time, $\tau$, lb |
| $W_f$ | fuel flow of unburdened engine, lb/hr |
| $W_{fc}$ | flexible coupling weight, lb |
| $W_h$ | weight of shaft housing, lb |
| $W_p$ | weight of pillow block, lb |
| $W_s$ | intermediate solid shaft weight, lb |
| $\alpha$ | adapter proportionality constant for shaft adapter weight |
| $\beta$ | secondary proportionality constant for pillow block weight, lb/in.$^2$ |
| $\omega$ | primary proportionality constant for pillow block weight, lb/in.$^2$ |
| $\varepsilon$ | ratio of housing clearance to outside diameter of the shaft $= c/D_o$ |
| $\eta$ | coupling unit efficiency |
| $\eta_{cs}$ | efficiency of constant speed device |
| $\gamma$ | weight density of torque tube material, lb/in.$^3$ |
| $\gamma_{ad}$ | weight density of adapter material, lb/in.$^3$ |
| $\gamma_h$ | weight density of shaft housing material, lb/in.$^3$ |
| $\gamma_p$ | weight density of pillow block material, lb/in.$^3$ |
| $\gamma_s$ | weight density of solid shaft material, lb/in.$^3$ |
| $\tau$ | duration of power extraction, hr |

$\Phi$ horsepower parameter defined by Eq. (V-8)

$\psi$ **specific thrust fuel consumption, lb/lb-hr, [see Eq. (III-4), Part 1]**

C. Derivations of Torque Parameter, $\Phi$, and the Solid Shaft Diameter, $D_s$

The horsepower extracted from the engine, $HP_x$, is given by

$$HP_x = \frac{HP_r}{\eta_{cs}\,\eta^n} \tag{V-1}$$

The first shaft must then transmit the following torque at rated speed:

$$T = \frac{HP_x\ 63000}{N} \tag{V-2}$$

where:

$T$ = continuous rated torque, lb-in.

$N$ = rated speed, rpm

It is required that the shaft be capable of transmitting a static torque equal to approximately 4.4 times the continuous torque (Ref. V-1 and V-2). From this, Eq. (V-2) becomes

$$T_s = 4.4\ T = \frac{4.4 \times 63000\ HP_x}{N} \tag{V-3}$$

Substituting from Eq. (V-1)

$$T_s = \frac{277{,}000\ \ HP_r}{N\ \ \eta_{cs}\,\eta^n} \tag{V-4}$$

where:

$T_s$ = static torque, lb-in.

From the fundamental equation for the shear stress in a hollow tube,

$$T_s = \frac{\pi\ s_s D_o^3\ (1 - K^4)}{16K_t} \tag{V-5}$$

where:

$s_s$ = design stress, psi

$K_t$ = shock factor

For suddenly applied loads, the value of the shock factor, $K_t$, will vary from 1.0 to 1.5 with minor shock, and from 1.5 to 3.0 with heavy shock (Ref. V-3).

Substituting for K and re-arranging, Eq. (V-5) can be written as

$$\frac{T_s K_t}{s_s} = 0.196\, D_o^3 \left[ 1 - \left( 1 - \frac{2t}{D_o} \right)^4 \right] \tag{V-6}$$

Multiplying Eq. (V-4) by $K_t/s_s$ and substituting in Eq. (V-6)

$$\frac{HP_r K_t}{N\, \eta_{cs} \eta^n s_s} = 0.708 \times 10^{-6}\, D_o^3 \left[ 1 - \left( 1 - \frac{2t}{D_o} \right)^4 \right] \tag{V-7}$$

By definition:

$$\Phi = \frac{HP_r K_t}{N\, \eta_{cs} \eta^n s_s} \tag{V-8}$$

Equation (V-7) is plotted in dotted lines on Fig. V-1. (Fig. V-3, Part 1) When the shaft is solid, $2t = D_o$ and Eq. (V-7) can be written as

$$D_s = 112\, (\Phi)^{1/3} \tag{V-9}$$

Equation (V-9) is shown graphically in Fig. V-2. (Fig. V-4, Part 1)

D. Derivation of Critical Speed Parameter, $L\sqrt{N_{cr}}$

For critical speed considerations each shaft unit, as indicated in Fig. V-3, is assumed to be uniformly loaded and simply supported. In the actual case the reduction from the torque tube to a solid shaft tends to decrease the stiffness of the shaft and, hence, decrease the critical speed. However, the stiffness is increased because the bearings are closer together than indicated by dimension L and because the bearings are not knife-edged supports. From these considerations it is believed that the original assumption gives a conservative critical speed. Torsional critical speeds have been neglected.

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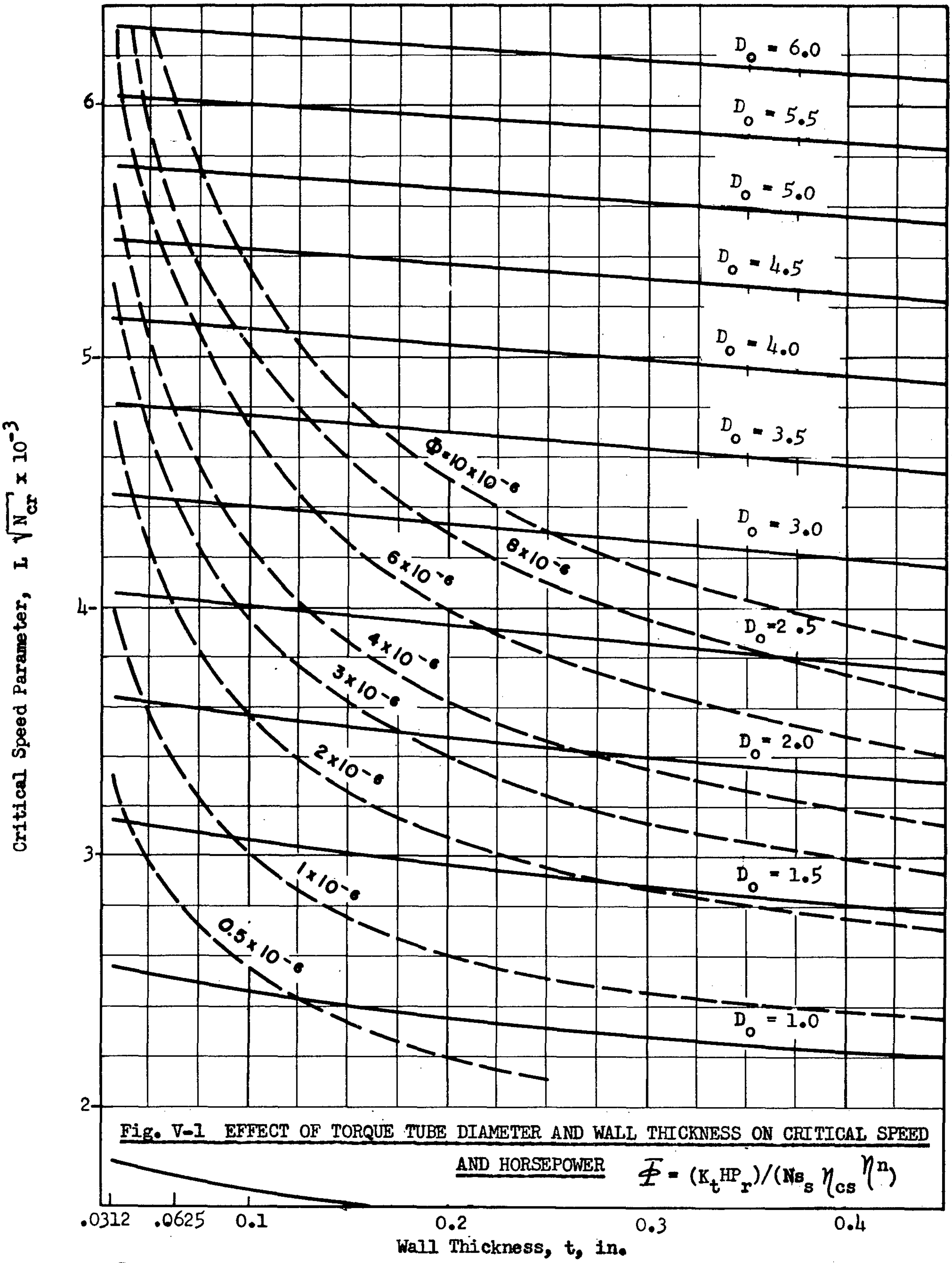


Fig. V-1 EFFECT OF TORQUE TUBE DIAMETER AND WALL THICKNESS ON CRITICAL SPEED AND HORSEPOWER $\Phi = (K_t HP_r)/(Ns_s \eta_{cs} \eta^n)$



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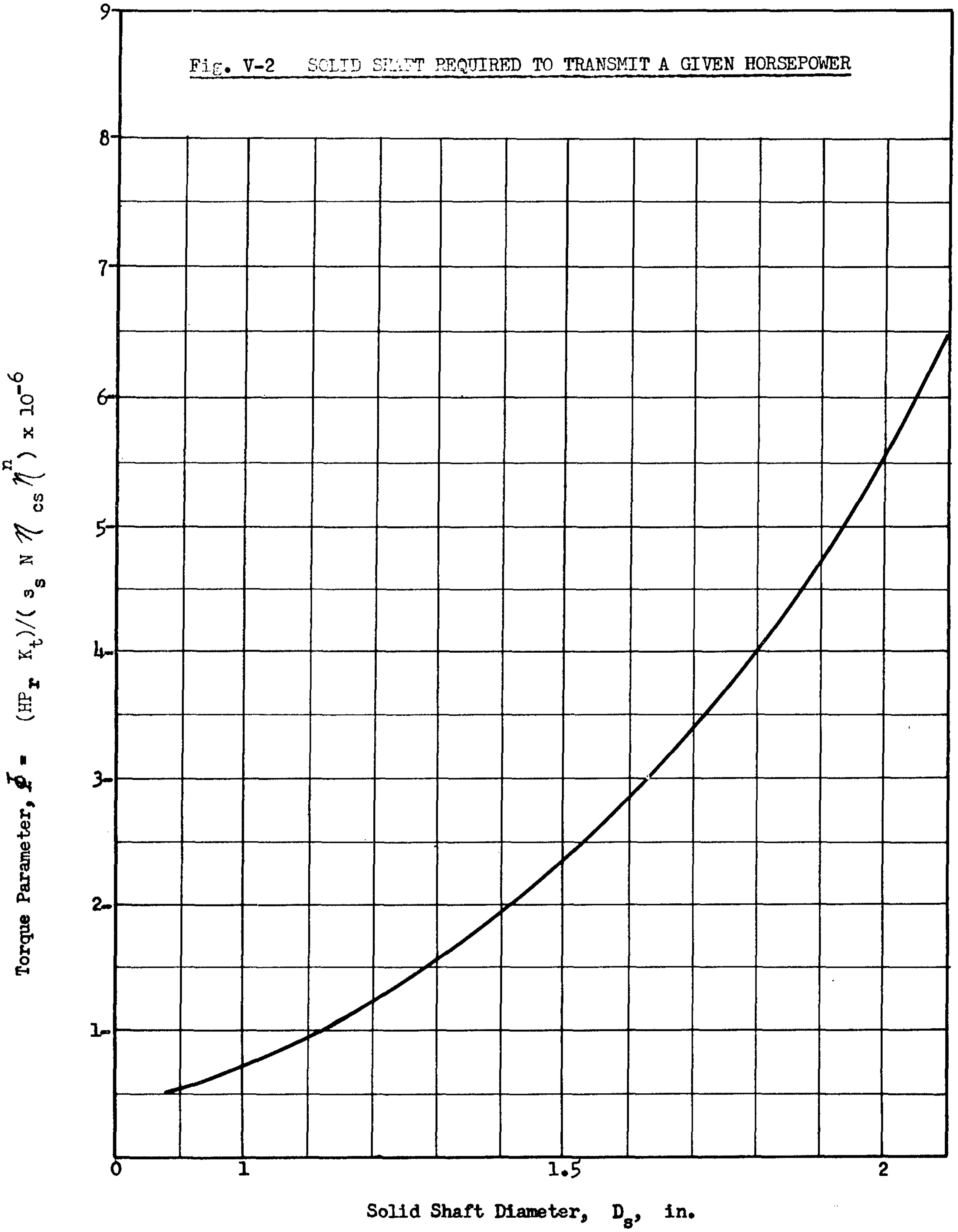



Fig. V-2 SOLID SHAFT REQUIRED TO TRANSMIT A GIVEN HORSEPOWER

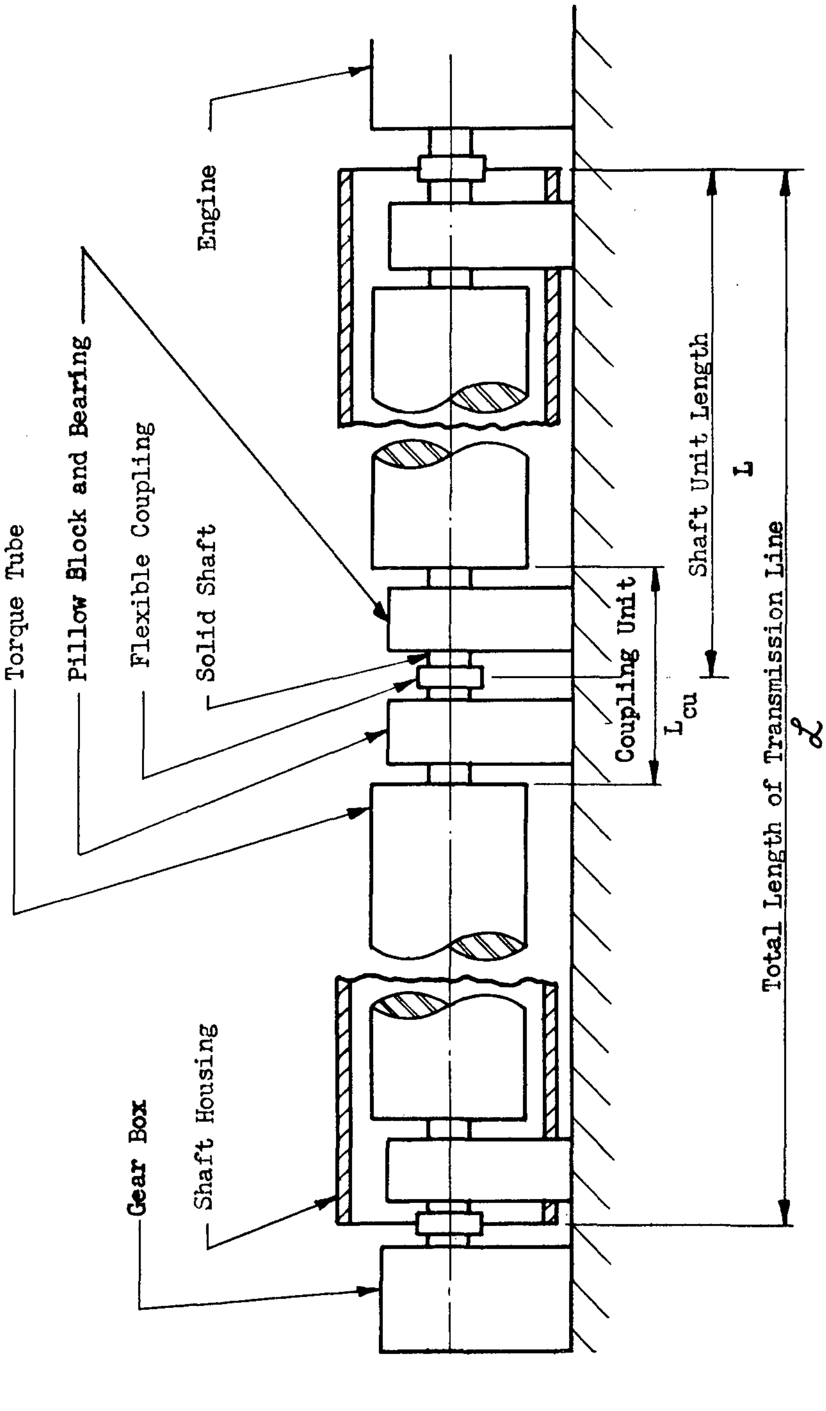



Fig. V-3 SCHEMATIC DIAGRAM OF MECHANICAL POWER TRANSMISSION SYSTEM

The fundamental equation for the critical speed of a uniformly loaded, simply supported shaft is given in Ref. V-3 as:

$$N_{cr} = \frac{60}{2\pi} \sqrt{\frac{\pi^4 E I g}{W L^3}} \qquad \text{(V-10)}$$

where:

$N_{cr}$ = critical speed, rpm

E = modulus of elasticity, lb/in.$^2$

I = rectangular moment of inertia, in.$^4$

g = acceleration due to gravity, in/sec$^2$

W = weight of the shaft, lb

L = distance between supports, in.

For a hollow tube,

$$I = \frac{\pi D_o^4}{64} ( 1 - K^4) \qquad \text{(V-11)}$$

$$W = \frac{\pi \gamma L D_o^2}{4} ( 1 - K^2 ) \qquad \text{(V-12)}$$

Substitute Eq. (V-11) and (V-12) in Eq. (V-10)

$$N_{cr} = 7.50\pi \sqrt{\frac{E g}{\gamma}} \frac{D_o}{L^2} \sqrt{1 + K^2} \qquad \text{(V-13)}$$

For a steel shaft ( $\gamma$ = 0.284 lb/in.$^3$)

$$N_{cr} = 47.5 \times 10^5 \frac{D_o}{L^2} (1 + K^2)^{1/2} \qquad \text{(V-14)}$$

Solving for L and expressing K in terms of $D_o$ and **t**, gives

$$L = \left[ \frac{47.5 \times 10^5 \; D_o}{N_{cr}} \sqrt{1 + \left[\frac{D_o - 2t}{D_o}\right]^2} \right]^{1/2} \qquad \text{(V-15)}$$

or

$$L\sqrt{N_{cr}} = \left[47.5 \times 10^5 \sqrt{D_o^2 + (D_o - 2t)^2}\right]^{1/2} \qquad \text{(V-16)}$$

Equation (V-16) is plotted as solid lines on Fig. V-1.

E. Weight of the Pillow Block, $W_p$

The weight of the pillow block is considered to be proportional to the outside bearing diameter squared. Under some conditions, the torque tube diameter may be large enough to require that the pillow block be raised to provide sufficient clearance between the torque tube and the frame of the airplane. This would require extra material and, hence, added weight on the pillow block. The weight of the pillow block may be approximated by the following equation:

$$W_p = \omega D_b^2 + \beta (D_o^2 - D_b^2) \frac{\pi}{4} \qquad \text{(V-17)}$$

The value of $\beta$ depends on the relative magnitude of $D_o$ and $D_b$ as shown below.

Case I: $D_o \leq 1.25 D_b$

Under this condition it is assumed that the standard dimensions of the pillow block will provide adequate clearance for the torque tube.

Therefore, $\beta = 0$, and $\omega$ is determined from manufacturer's data. (See Fig. V-4a.)

Case II: $D_o > 1.25 D_b$

Under this condition it is assumed that extra material must be added to the pillow block to provide adequate clearance for the torque tube.

(See Fig. V-4b). Thus $\beta$ takes on a value greater than zero.

$$\beta = R \gamma_p w_p \qquad \text{(V-18)}$$

where:

$\gamma_p$ = weight density of pillow block material, lb/in.$^3$

$w_p$ = width of the pillow block, in.

Case I: $D_o \leqq 1.25\, D_b$

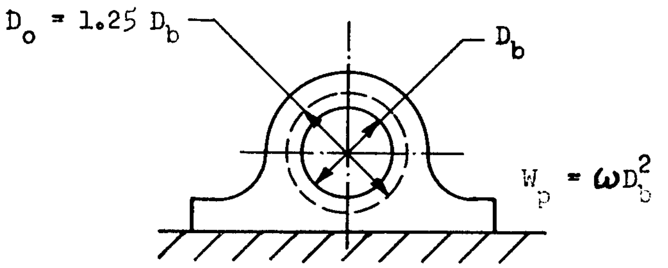



Fig. V-4a

Case II: $D_o > 1.25\, D_b$

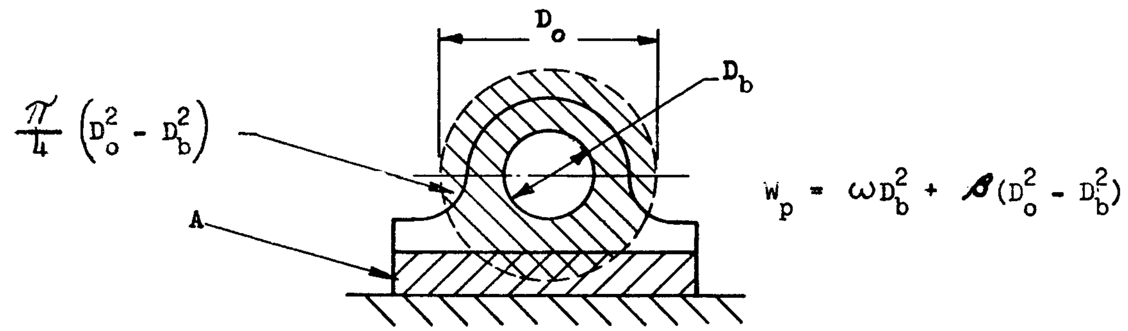



Fig. V-4b

Fig. V-4 SCHEMATIC DIAGRAM OF PILLOW BLOCK CONFIGURATIONS

R is the ratio of the cross-sectional area which must be added to a standard pillow block to provide clearance for the torque tube and the annular area between the outside diameter of the bearing and the outside diameter of the torque tube.

$$R = \frac{4A}{\pi (D_o^2 - D_b^2)}$$

The value of R is a function of the particular style of pillow block and must be obtained from the design of the particular pillow block. (See Fig. V-4.)

F. <u>Weight of Shaft per Unit Length,</u> $\frac{W}{L}$

From the basic equation for the weight of a hollow shaft,

$$\frac{W}{L} = \frac{\pi \gamma D_o^2}{4} (1 - K^2) \qquad \text{(V-19)}$$

where:

$\frac{W}{L}$ = weight per unit length of torque tube, lb/in.

$\gamma$ = weight density of material, lb/in.$^3$

$K$ = diameter ratio = $D_i/D_o$

The term $(1 - K^2)$ can be written as

$$(1 - K^2) = 1 - \frac{(D_o - 2t)^2}{D_o^2} \qquad \text{(V-20)}$$

or

$$(1-K^2) = \frac{4t(D_o - t)}{D_o^2} \qquad \text{(V-21)}$$

where:

$t$ = wall thickness of tube, in.

Substitute Eq. (V-21) in Eq. (V-19),

$$\frac{W}{L} = \pi \gamma t(D_o - t) \qquad \text{(V-22)}$$

Equation (V-22) is shown graphically in Fig. V-5.

## G. Weight of Fuel, $W_F$

The weight of fuel required to operate the transmission system is obtained from

$$W_F = C''_{PX}\, HP_x\, \tau \tag{V-23}$$

where

$C''_{PX}$ = specific fuel consumption of the transmission system, lb/HP-hr

$HP_x$ = horsepower extracted from the engine

$\tau$ = duration of power extraction, hr

The extracted horsepower may now be expressed as:

$$HP_x = \frac{HP_r}{\eta_{cs}\ \eta^n} \tag{V-24}$$

where

$HP_r$ = rated power output of transmission system

$\eta_{cs}$ = efficiency of the constant speed drive

$\eta$ = coupling unit efficiency

$n$ = number of shaft units

The specific fuel consumption of the transmission system is found from

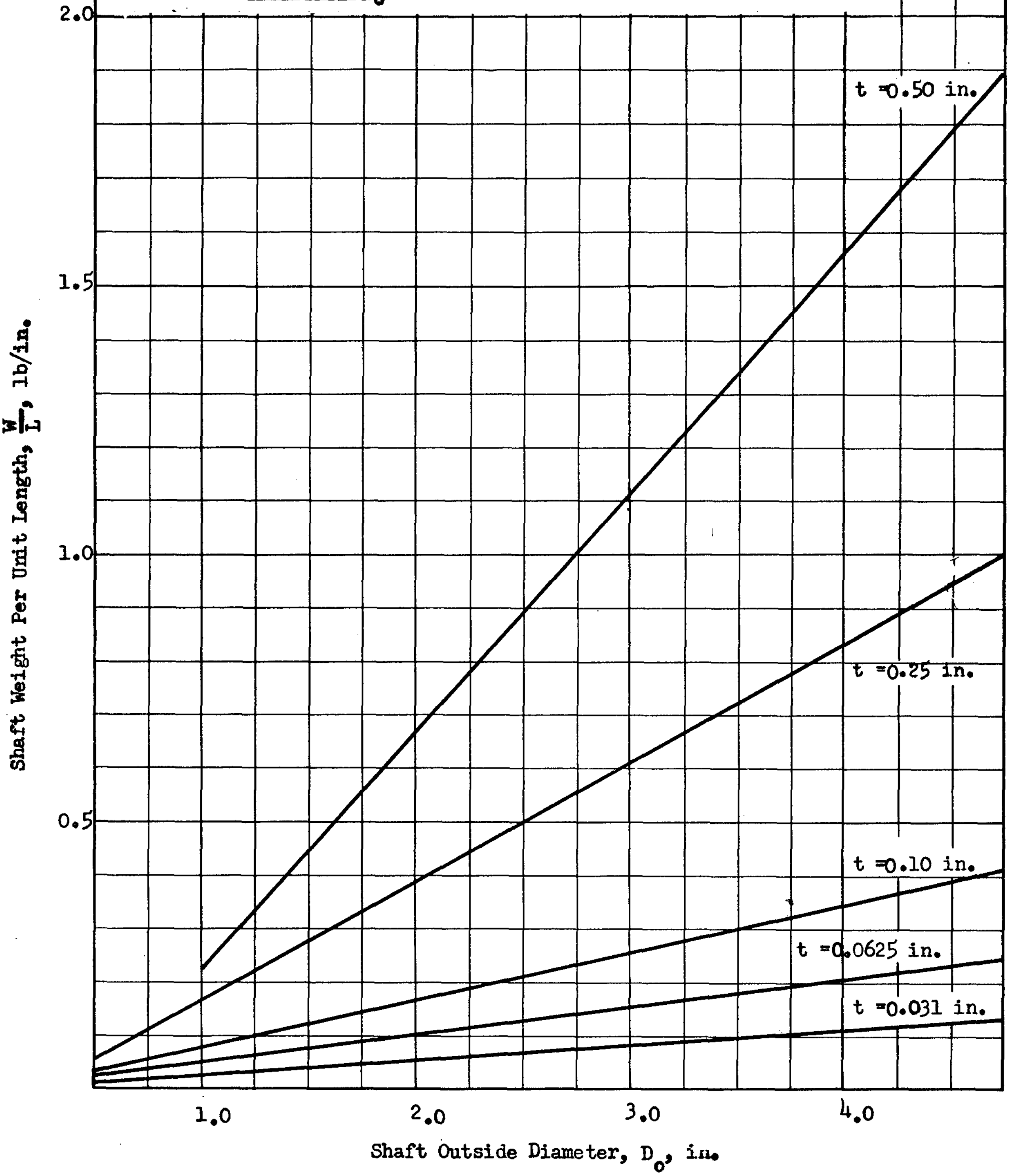
$$C''_{PX} = \frac{\psi\ F_n\ C_{PX} - W_F\ C'_{PX}}{HP_{REF}} \tag{V-25}$$

where

$C''_{PX}$ = specific fuel consumption of the transmission system, lb/$HP_x$hr

$F_n$ = thrust of unburdened engine, lb

**Fig. (V-5) WEIGHT OF SHAFT PER UNIT LENGTH, $\frac{W}{L}$, VS OUTSIDE DIAMETER, $D_o$**



$\psi$ = specific thrust fuel consumption, lb/lb-hr [See Eq. (III-4), Part 1]

$C'_{PX}$ = fuel flow correction factor due to power extraction

$C_{PX}$ = thrust correction factor due to power extraction

$W_f$ = fuel flow of unburdened engine, lb/hr

$HP_{REF}$ = reference HP, arbitrarily taken as 10 per cent of jet horsepower at sea level and stationary conditions

All of the factors in Eq. (V-25) can be obtained from engine specifications published by the engine manufacturer.

Equation (V-25) is derived in detail in Section III of Part 2. Equation (V-23) can now be written as:

$$W_F = \frac{C''_{PX}\, HP_r\, \tau}{\eta_{cs}\, \eta^n} \qquad \text{(V-26)}$$

H. Weight of Shaft Housing, $W_h$

When a housing is required for the shaft, it is assumed that the housing covers the entire shaft system with only minor breaks and interruptions for fastening to the frame or mounting, the weight of the bolts and fittings being equivalent to the weight of metal cut out for mounting.

$$W_h = 12\pi\, \gamma_h\, t_h\, D_h\, \mathcal{L} \qquad \text{(V-27)}$$

where:

$W_h$ = weight of housing, lb

$\gamma_h$ = weight density of housing material, lb/in.$^3$

$t_h$ = thickness of housing, in.

$\mathcal{L}$ = total length of housing, ft.

$D_h$ = inside diameter of housing, in. = $D_o\,(1 + 2\varepsilon)$

$\varepsilon$ = $\frac{c}{D_o}$ (assumed constant)

c = clearance between shaft and housing, in.

Then,

$$W_h = 37.7\,\gamma_h t_h \mathcal{L}\,(1 + 2\,\varepsilon)\ D_o \qquad \text{(V-28)}$$

## I. Approximate Optimum Shaft Diameter, $(D_o)_{opt}$

To obtain the optimum shaft diameter, the total weight equation is differentiated with respect to $D_o$. The resulting expression is equated to zero and solved for $D_o$. This gives the value of $D_o$ which will minimize the total weight of the system.

The total weight of the system is given by:

$$\Sigma W = n \left[ \frac{W}{L} L + W_{cu} \right] + W_F + W_h + W_{cs} \qquad \text{(V-29)}$$

where:

- $n$ = number of shaft units
- $W$ = weight of a shaft unit, lb
- $L$ = length of shaft unit, in.
- $W_{cu}$ = weight of coupling unit, lb
- $W_F$ = weight of fuel required to operate the accessory system for a given time, $\tau$ , lb
- $W_h$ = weight of shaft housing, lb
- $W_{cs}$ = weight of the constant speed drive, lb

Each of the terms in Eq. (V-29) must be expressed in terms of $D_o$ for differentiation. This is done by means of the following approximations:

1. Approximation for Shaft Length

Equation (V-13) expresses the critical speed as follows:

$$N_{cr} = 7.50 \pi \sqrt{\frac{E\ g}{\gamma}} \frac{D_o}{L^2} \sqrt{1 + K^2}$$

Let,

$$Z = 7.50 \pi \left( \frac{E\ g}{\gamma} \right)^{1/2} \qquad \text{(V-30)}$$

Then,

$$N_{cr} = Z \frac{D_o}{L^2} \sqrt{1 + K^2} \qquad \text{(V-31)}$$

and

$$L = \left[ \frac{Z}{N_{cr}} \; D_o \sqrt{1 + K^2} \right]^{1/2} \qquad (V\text{-}32)$$

$$(1 + K^2)^{1/2} = \left[ 1 + \frac{(D_o - 2t)^2}{D_o^2} \right]^{1/2} \qquad (V\text{-}33)$$

Expanding,

$$(1 + K^2)^{1/2} = \left[ 2 - \frac{4t}{D_o} + \frac{4t^2}{D_o^2} \right]^{1/2} \qquad (V\text{-}34)$$

Assuming $t$ is small compared to $D_o$, the last term in Eq. (V-34) can be neglected. Dropping the last term and factoring gives

$$(1 + K^2)^{1/2} = \sqrt{2} \left[ 1 - \frac{2t}{D_o} \right]^{1/2} \qquad (V\text{-}35)$$

Expanding in a binominal series, and using only the first two terms

$$(1 + K^2)^{1/2} = \sqrt{2} \left[ 1 - \frac{t}{D_o} \right] \qquad (V\text{-}36)$$

Substitute Eq. (V-36) in Eq. (V-32)

$$L = \left[ \frac{\sqrt{2}\, Z}{N_{cr}} \left( 1 - \frac{t}{D_o} \right) \right]^{1/2} (D_o)^{1/2} \qquad (V\text{-}37)$$

Let

$$Q = \left[ \frac{\sqrt{2}\, Z}{N_{cr}} \left( 1 - \frac{t}{D_o} \right) \right]^{1/2} \qquad (V\text{-}38)$$

By plotting $L$ vs $(D_o)^{1/2}$, it was found that $Q$ remained comparatively constant over the range of $D_o$ compatible with the current state of aircraft accessory development.

Thus, L is a function of $D_o$,

$$L = Q(D_o)^{1/2} \qquad (V\text{-}39)$$

2. <u>Approximation for Shaft Weight</u>

For a thin walled tube the weight can be approximated by

$$\frac{W}{L} = \pi \gamma t D_o \tag{V-40}$$

3. <u>Approximation for Coupling Unit Weight</u>

The coupling unit weight is the sum of the component weights, (See Fig. V-6).

$$W_{cu} = 2W_{ad} + 2W_p + 2W_b + W_{fc} + W_s \tag{V-41}$$

where:

$W_{ad}$ = weight of the adapter, lb

$W_p$ = weight of the pillow block, lb

$W_b$ = weight of the bearing, lb (from manufacturer's data)

$W_{fc}$ = weight of the flexible coupling, lb (from manufacturer's data)

$W_s$ = weight of solid, intermediate shaft, lb

From Eqs. (V-11), (V-12), (V-13) of Part 1

$$W_{cu} = 2\alpha(D_o^2 - D_s^2) + 2\left[\beta(D_o^2 - D_b^2) + \omega D_b^2\right] + 2W_b + W_{fc} + \left(\frac{\pi \gamma_s L_s D_s^2}{4} - \frac{W}{L} L_{cu}\right) \tag{V-42}$$

Regrouping the terms,

$$W_{cu} = 2(\alpha+\beta) D_o^2 - 2\left[\alpha D_s^2 + (\beta-\omega) D_b^2\right] + 2W_b + W_{fc} + \left(\frac{\pi \gamma_s L_s D_s^2}{4} - \frac{W}{L} L_{cu}\right) \tag{V-43}$$

Collect the constant terms by letting

$$\delta = 2(\alpha+\beta) \tag{V-44}$$

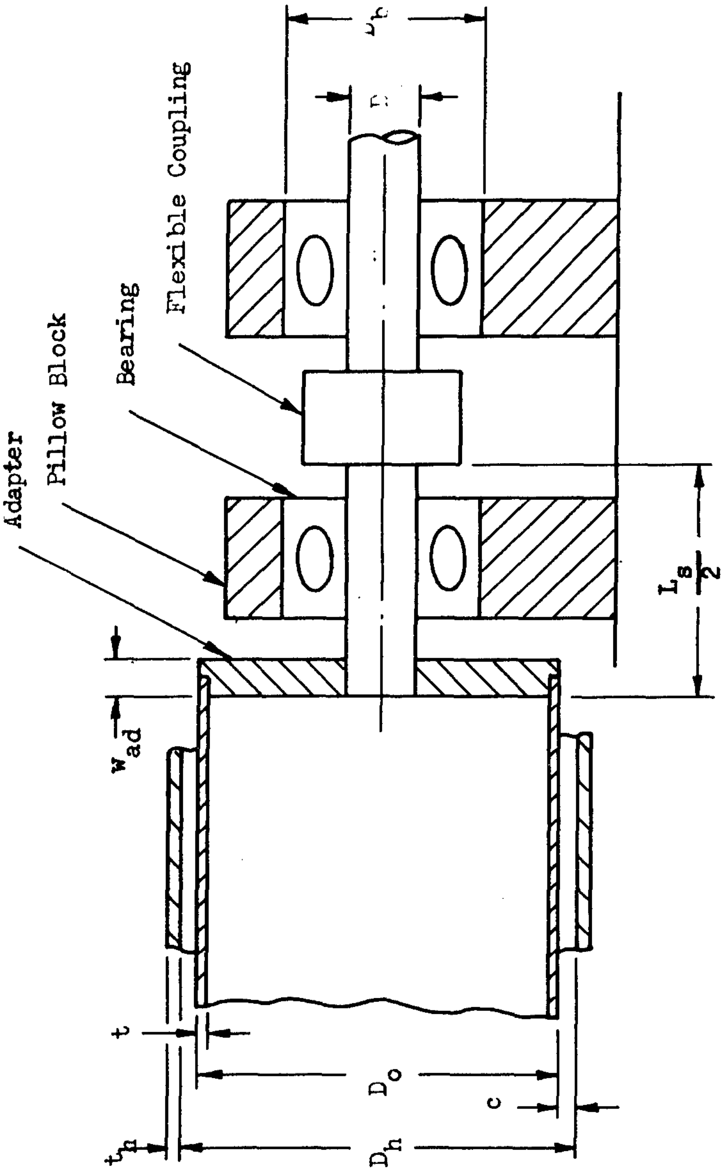



Fig. V-6 SCHEMATIC DIAGRAM OF COUPLING UNIT COMPONENTS

and

$$\bar{E} = 2W_b + W_{fc} + \frac{\pi \gamma_s L_s D_s^2}{4} - \frac{W}{L} L_{cu} - 2\left[\alpha D_s^2 + (\beta - \omega) D_b^2\right] \qquad \text{(V-45)}$$

Then,

$$W_{cu} = \delta D_o^2 + \bar{E} \qquad \text{(V-46)}$$

The total weight equation can now be written as follows by substituting Eqs. (V-39), (V-40), (V-46), (V-26) and (V-28) in Eq. (V-29).

$$\Sigma W = \frac{\mathcal{L}}{Q\sqrt{D_o}}\left[\pi\gamma t\, D_o\, (Q\sqrt{D_o}) + \delta D_o^2 + \bar{E}\right] + \frac{C''_{PX}\, HP_o\, \tau}{\eta_{cs}} (\eta)^{-\mathcal{L}/(Q\sqrt{D_o})}$$

$$+ \pi\gamma_h t_h (1 + 2\varepsilon)\, \mathcal{L} D_o + W_{cs} \qquad \text{(V-47)}$$

Let

$$x = \sqrt{D_o} \qquad \text{(V-48)}$$

Substituting Eq. (V-48) in Eq. (V-47) and collecting terms

$$\Sigma W = \frac{\mathcal{L}\,\delta}{Q} x^3 + \pi\mathcal{L}\left[\gamma t + \gamma_h t_h (1 + 2\varepsilon)\right] x^2 + \frac{\mathcal{L}\,\bar{E}}{Q\,x}$$

$$+ \frac{C''_{PX}\, HP_r\, \tau}{\eta_{cs}} (\eta)^{-\mathcal{L}/(Q\,x)} + W_{cs} \qquad \text{(V-49)}$$

Differentiating Eq. (V-49) with respect to $x$ and equating to zero yields:

$$\frac{\partial \Sigma W}{\partial x} = 3\frac{\mathcal{L}\,\delta}{Q} x^2 + 2\pi\mathcal{L}\left[\gamma t + \gamma_h t_h (1 + 2\varepsilon)\right] x$$

$$- \left[\frac{\mathcal{L}\,\bar{E}}{Q} - \frac{C''_{PX}\, HP_r\, \tau}{\eta_{cs}} \quad \frac{\mathcal{L} \ln \eta}{Q\,\eta^n}\right] \frac{1}{x^2} = 0 \qquad \text{(V-50)}$$

Let

$$C_1' = 3\,\frac{\mathcal{L}\,\delta}{Q} \tag{V-51}$$

$$C_2' = 2\,\pi\mathcal{L}\left[\gamma t + \gamma_h t_h\,(1 + 2\,\varepsilon)\right] \tag{V-52}$$

$$C_3' = \frac{\mathcal{L}\,\bar{E}}{Q} - \frac{C''_{PX}\,HP_r\,\tau\,\mathcal{L}\,\ln\eta}{Q\,\eta_{cs}\,\eta^{n}} \tag{V-53}$$

Substituting Eq. (V-51), (V-52) and (V-53) in Eq. (V-50) and multiplying by $x^2$

$$C_1'\,x^4 + C_2'\,x^3 - C_3' = 0 \tag{V-54}$$

Let

$$C_1 = \frac{C_1'}{C_3'} \tag{V-55}$$

and

$$C_2 = \frac{C_2'}{C_3'} \tag{V-56}$$

Then,

$$C_1\,x^4 + C_2\,x^3 - 1 = 0 \tag{V-57}$$

Substituting Eq. (V-48) in Eq. (V-56) gives

$$C_2(D_o)^{3/2} = 1 - C_1\,D_o^2 \tag{V-58}$$

Equation (V-58) is solved graphically in Fig. V-7. The value of $D_o$ which satisfies this equation is the optimum shaft diameter, $(D_o)_{opt}$ and it is found at the intersection of the two integral curves drawn for the calculated values of $C_1$ and $C_2$.

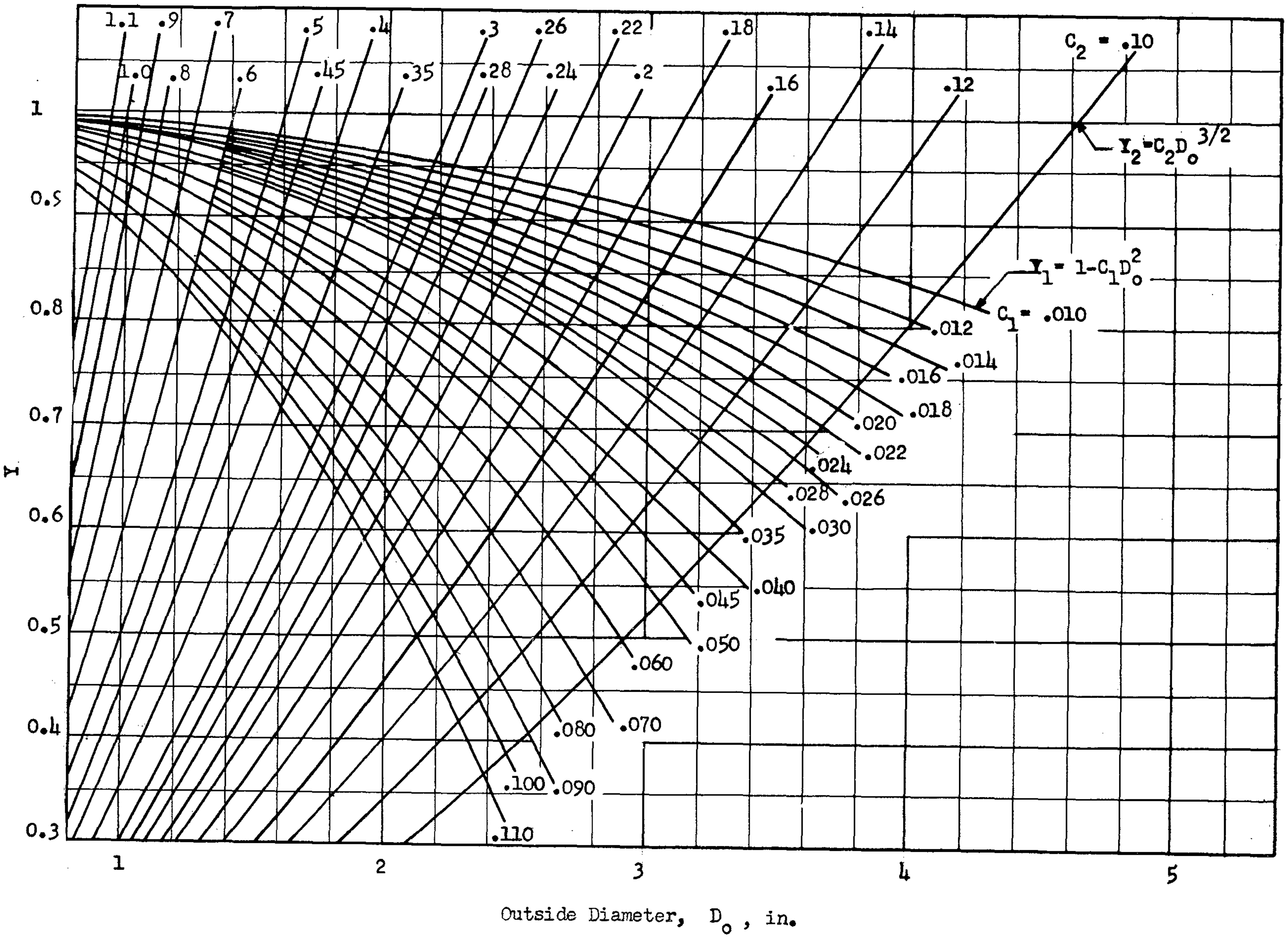



Fig. V-7 GRAPHICAL SOLUTION OF $C_2 D_o^{3/2} = 1 - C_1 D_o^2$

By making appropriate substitutions in the above equations, the parameters $C_1$ and $C_2$ become

$$C_1 = \frac{6(\alpha + \beta)\eta^n}{\left\{ 2W_b + W_{fc} + \left( \frac{\pi \gamma_s L_s D_s^2}{4} - \frac{W}{L} L_{cu} \right) - 2\left[ \alpha D_s^2 + (\beta - \omega) D_b^2 \right] \right\} \eta^n - \frac{C''_{PX}\, HP_r\, \tau \ln \eta}{\eta_{cs}}}$$

(V-59)

$$C_2 = \frac{136.9\, \eta^n \sqrt{1 - \frac{t}{D_o}} \left[ \gamma t + \gamma_h t_h (1 + 2\varepsilon) \right]}{\left\{ 2W_b + W_{fc} + \left( \frac{\pi \gamma_s L_s D_s^2}{4} - \frac{W}{L} L_{cu} \right) - 2\left[ \alpha D_s^2 + (\beta - \omega) D_b^2 \right] \right\} \eta^n - \frac{C''_{PX}\, HP_r\, \tau \ln \eta}{\eta_{cs}}}$$

(V-60)

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## WEIGHT ANALYSIS OF A SINGLE-STEP SPEED REDUCER

Section VI

### A. Introduction

The equations and curves for the following factors are derived and explained in this section:

1. Solid Shaft Diameter
2. Relationship between $d_s$ and $d_o$ for Equal Strength Shafts
3. Minimum Number of Pinion Teeth for a Given Gear Ratio
4. Weight Factor
5. Projected Areas for Gear Housing Weight

### B. Nomenclature

| | |
|---|---|
| A | projected area perpendicular to the centerline of the gears, $in.^2$ |
| a | weight factor |
| C | height of gear box, in. |
| c | hole factor |
| D | pitch diameter, in. |
| d | diameter, in. |
| F | tangential force at pitch line, lb |
| F(R) | function defined by Eq. (VI-42) |
| f | face width of the gears, in. |
| h | spline addendum, in. |
| HP | horsepower |
| j | scale factor |
| $K_t$ | shock factor for shaft |
| $K_w$ | housing width factor, in. |
| L | length of line tangent to outside diameter of both gears, in. |

| | |
|---|---|
| m | minimum thickness of material above keyway, in. |
| N | speed, rpm |
| n | number of gear teeth |
| P | diametral pitch, 1/in. |
| R | gear reduction ratio |
| S | number of spline teeth |
| (SF) | service factor for gears |
| $s_w$ | working stress for gear teeth, psi |
| $s_s$ | design stress for shaft, psi |
| T | torque, lb-in. |
| t | thickness, in. |
| V | pitch line velocity, fpm |
| W | weight, lb |
| w | width of gear housing, in. |
| Y | form factor for Lewis Equation |
| z | diametral pitch x addendum |
| $\gamma$ | weight density, lb/in.$^3$ |
| $\Theta$ | angle between line through gear centers and a line perpendicular to common tangent of the outside diameters of the gears, degrees |
| $\Phi$ | pressure angle of gears, degrees |

SUBSCRIPTS

| | |
|---|---|
| 1 | input |
| 2 | output |
| ave | average |
| e | equivalent |
| G | gears (both pinion and large gear) |

| | |
|---|---|
| H | hub |
| h | housing |
| i | inside |
| L | large gear |
| o | outside |
| p | pinion |
| pd | pad |
| r | rim |
| sp | spline |
| t | total |
| w | web |

C. <u>Solid Shaft Diameter</u>

It is desired to determine the solid shaft diameter required to transmit a given horsepower at a specified speed.

From the basic equation for shear stress in a round shaft under torsion

$$K_t T = \frac{\pi s_s d_s^3}{16} \qquad \text{(VI-1)}$$

where:

$T$ = torque, lb-in.

$K_t$ = shock factor

$s_s$ = design shear stress, psi

$d_s$ = shaft diameter, in.

The value of the shock factor, $K_t$, will vary from 1.0 to 1.5 for suddenly applied loads with minor shock, and from 1.5 to 3.0 for suddenly applied loads with heavy shock. (Ref. VI-1)

From the fundamental horsepower equation,

$$T = 5250 \times 12 \frac{HP}{N} \text{ lb-in.} \qquad \text{(VI-2)}$$

Substituting:

$$\frac{HP\ K_t}{N\ s_s} = 3.12 \times 10^{-6}\ d_s^3 \tag{VI-3}$$

This equation is shown graphically in Fig. VI-1.

D. <u>Relation Between $d_s$ and $d_o$ for Equal Strength Shafts</u>

Given a solid shaft splined to a hollow shaft of equal strength, it is **desired** to determine the relationship between solid shaft diameter and the outside diameter of the hollow shaft.

For both shafts to have equivalent strength, the maximum stress in the solid shaft must be equal to the maximum stress in the hollow shaft. From elementary strength of materials:

For a solid shaft:

$$s_s = \frac{16T}{\pi d_s^3} \tag{VI-4}$$

where:

$s_s$ = maximum shear stress, psi

$T$ = torque, lb-in.

$d_s$ = diameter of solid shaft, in.

For a hollow shaft:

$$s_s = \frac{16T}{\pi d_o^3 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]} \tag{VI-5}$$

where:

$d_o$ = outside diameter, in.

$d_i$ = inside diameter, in.

Equating Eqs. (VI-4) and (VI-5) gives:

$$\left[\frac{d_o}{d_s}\right]^3 = \frac{d_o^4}{d_o^4 - d_i^4} \tag{VI-6}$$

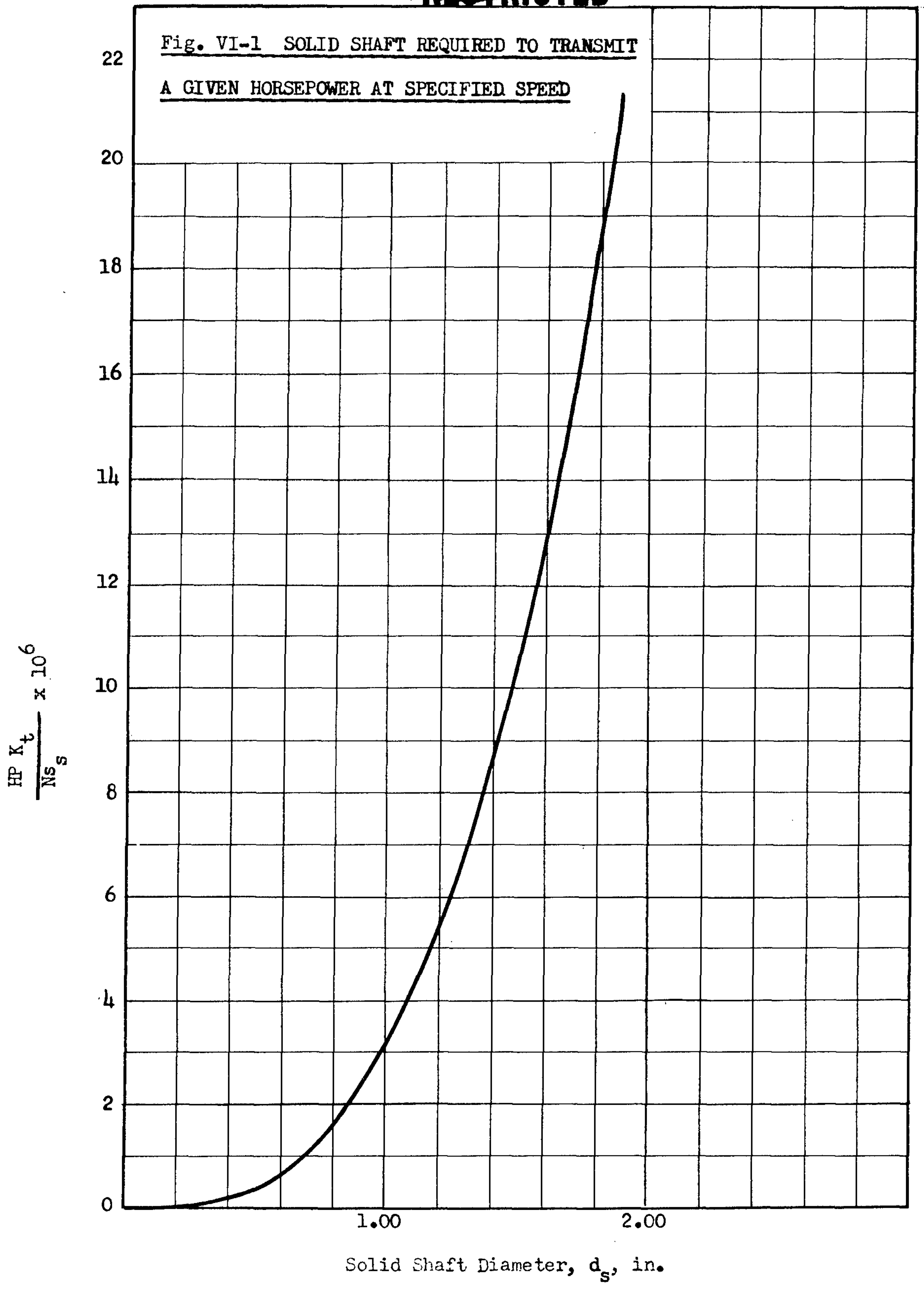


Fig. VI-1 SOLID SHAFT REQUIRED TO TRANSMIT A GIVEN HORSEPOWER AT SPECIFIED SPEED

The diametral pitch of the spline is defined as:

$$P_{sp} = \frac{S}{D_{sp}} \tag{VI-7}$$

where,

$P_{sp}$ = diametral pitch of spline, 1/in.

$S$ = number of teeth in spline

$D_{sp}$ = spline pitch diameter, in.

For this investigation it is assumed that the spline addendum and dedendum are equal. Also, it is assumed that the minor diameter of the external spline is equal to the solid shaft diameter, and the major diameter of the internal spline is equal to the inside diameter of the hollow shaft. This is shown in Fig. VI-2.

Then Eq. (VI-7) can be written as:

$$P_{sp} = \frac{S}{d_s + 2h} \tag{VI-8}$$

and

$$h = \frac{d_i - d_s}{4} \tag{VI-9}$$

Substituting in Eq. (VI-8)

$$P_{sp} = \frac{2S}{d_s + d_i} \tag{VI-10}$$

According to S. A. E. spline standards:

$$d_i = \frac{S + 1.8}{P_{sp}} \tag{VI-11}$$

Solving for $P_{sp}$ and equating to Eq. (VI-10) gives:

$$\frac{S + 1.8}{d_i} = \frac{2S}{d_s + d_i} \tag{VI-12}$$

or

$$d_i = d_s \left[\frac{S + 1.8}{S - 1.8}\right] \tag{VI-13}$$

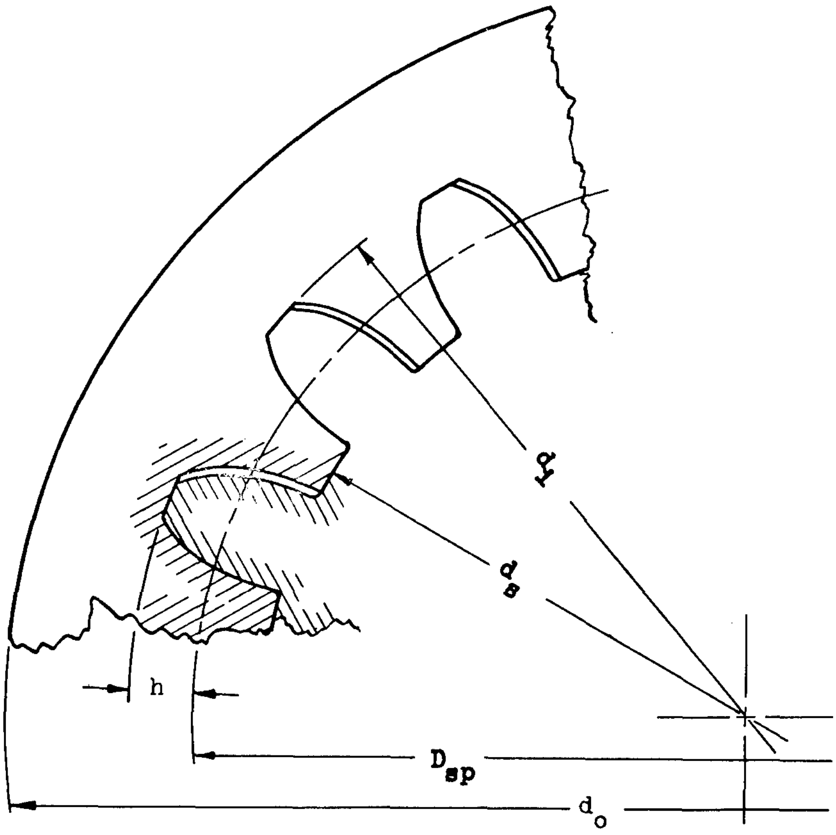


**Fig. VI-2 SPLINE DETAIL**

Substituting in Eq. (VI-6)

$$\left[\frac{d_o}{d_s}\right]^3 = \frac{\left[d_o \quad (S - 1.8)\right]^4}{\left[\ d_o \quad (S - 1.8)\right]^4 - \left[d_s \ (S + 1.8)\right]^4} \qquad \text{(VI-14)}$$

Rearranging and simplifying gives

$$\left[\frac{d_o}{d_s}\right]^4 - \frac{d_o}{d_s} = \left[\frac{S + 1.8}{S - 1.8}\right]^4 \qquad \text{(VI-15)}$$

The relationship between $\frac{d_o}{d_s}$ and S given in this equation is shown graphically in Fig. VI-3.

E. Minimum Number of Pinion Teeth for Given Gear Ratio

The geometry of involute gears is such that if too small a pinion (with an insufficient number of teeth) is used with a given gear, interference will occur between the addendum of the pinion teeth and the radial portion or flank of the mating gear teeth. With these considerations in mind, the following relationship is derived in Ref. VI-1:

$$n_p^2 + 2n_p n_L = \frac{4z(n_L + z)}{\sin^2 \Phi} \qquad \text{(VI-16)}$$

where:

$n_p$ = minimum number of teeth on pinion which will operate without interference with a given gear

$n_L$ = largest number of teeth on the large gear which will operate without interference with a given pinion

$\Phi$ = pressure angle, deg

z = (P) x (addendum).

The gear ratio is defined as:

$$R = \frac{n_L}{n_p} \qquad \text{(VI-17)}$$

Substituting for $n_L$ in Eq. (VI-16) and assuming the addendum equal to 1/P:

$$n_p^2 + 2Rn_p^2 = \frac{4\ (Rn_p + 1)}{\sin^2 \Phi} \qquad \text{(VI-18)}$$

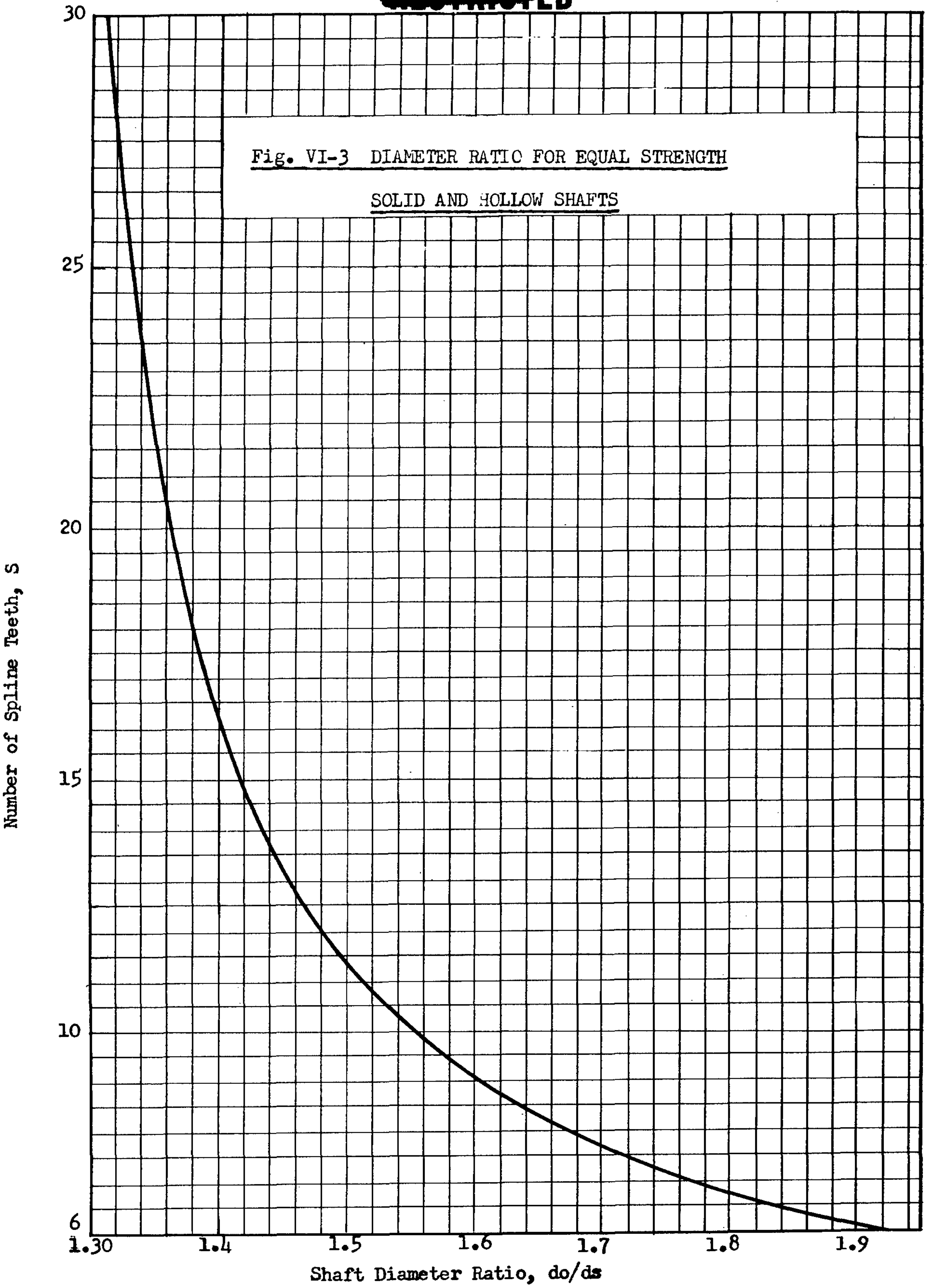


Fig. VI-3 DIAMETER RATIO FOR EQUAL STRENGTH SOLID AND HOLLOW SHAFTS

Equation (VI-18) can be written as:

$$n_p^2 \left(\sin^2\Phi + 2R \sin^2\Phi\right) - 4Rn_p - 4 = 0 \qquad \text{(VI-19)}$$

Substituting in the quadratic equation gives:

$$n_p = \frac{4R + \sqrt{16R^2 + 16 \sin^2\Phi (1 + 2R)}}{2 \sin^2\Phi (1 + 2R)} \qquad \text{(VI-19)}$$

This equation is shown graphically in Fig. (VI-4) for 20 degree full depth teeth.

## F. Weight Factor

The weight factor of a gear is defined as the ratio of the weight of a webbed gear with lightening holes to the weight of a solid gear. Figure VI-5 shows a schematic diagram of a typical gear. Using the notation as indicated in the diagram it is seen that

$$a = \frac{\frac{\pi}{4} \gamma_G \left[(D^2 - d_r^2)\ f + ct_w\ (d_r^2 - d_H^2) + f(d_H)^2\right]}{\frac{\pi}{4} \gamma_G\ f\ D^2} \qquad \text{(VI-20)}$$

where:

c = hole factor, ratio of gear web weight with holes to weight solid gear web. Factoring Eq. (VI-20) gives:

$$a = \left[1 - \left(\frac{d_r}{D}\right)^2\right] + \frac{ct_w}{f}\left[\left(\frac{d_r}{D}\right)^2 - \left(\frac{d_H}{D}\right)^2\right] + \left(\frac{d_H}{D}\right)^2 \qquad \text{(VI-21)}$$

The following proportions, obtained by empirical methods, are taken from Ref. VI-2. Referring to Fig. VI-5, the rim thickness, $t_r$, is found from

$$t_r = 0.5\pi \frac{1}{P} \qquad \text{(VI-22)}$$

The web thickness is determined from

$$t_w = 0.5\pi \frac{1}{P} + 0.125 \qquad \text{(VI-23)}$$

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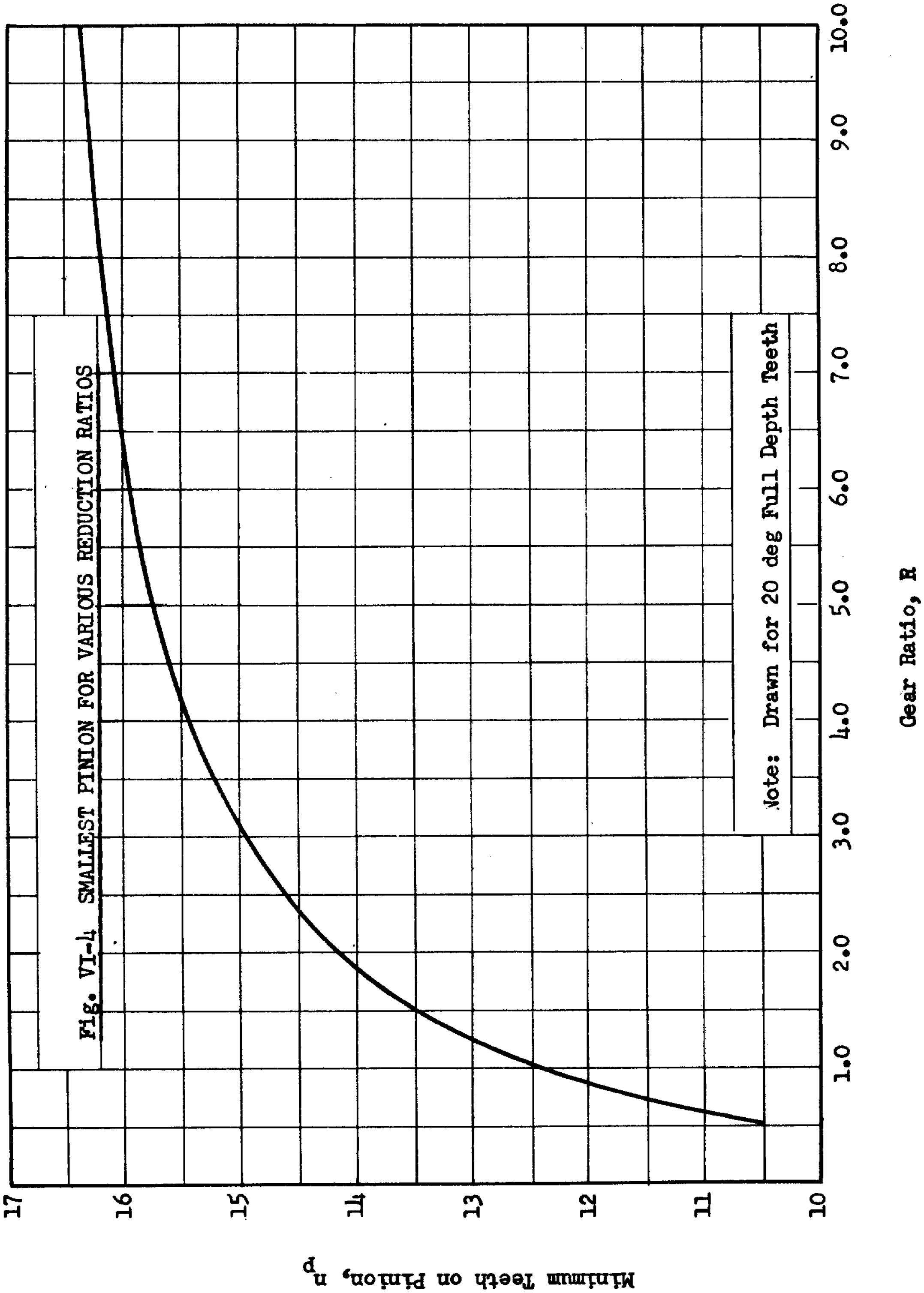


Fig. VI-4 SMALLEST PINION FOR VARIOUS REDUCTION RATIOS



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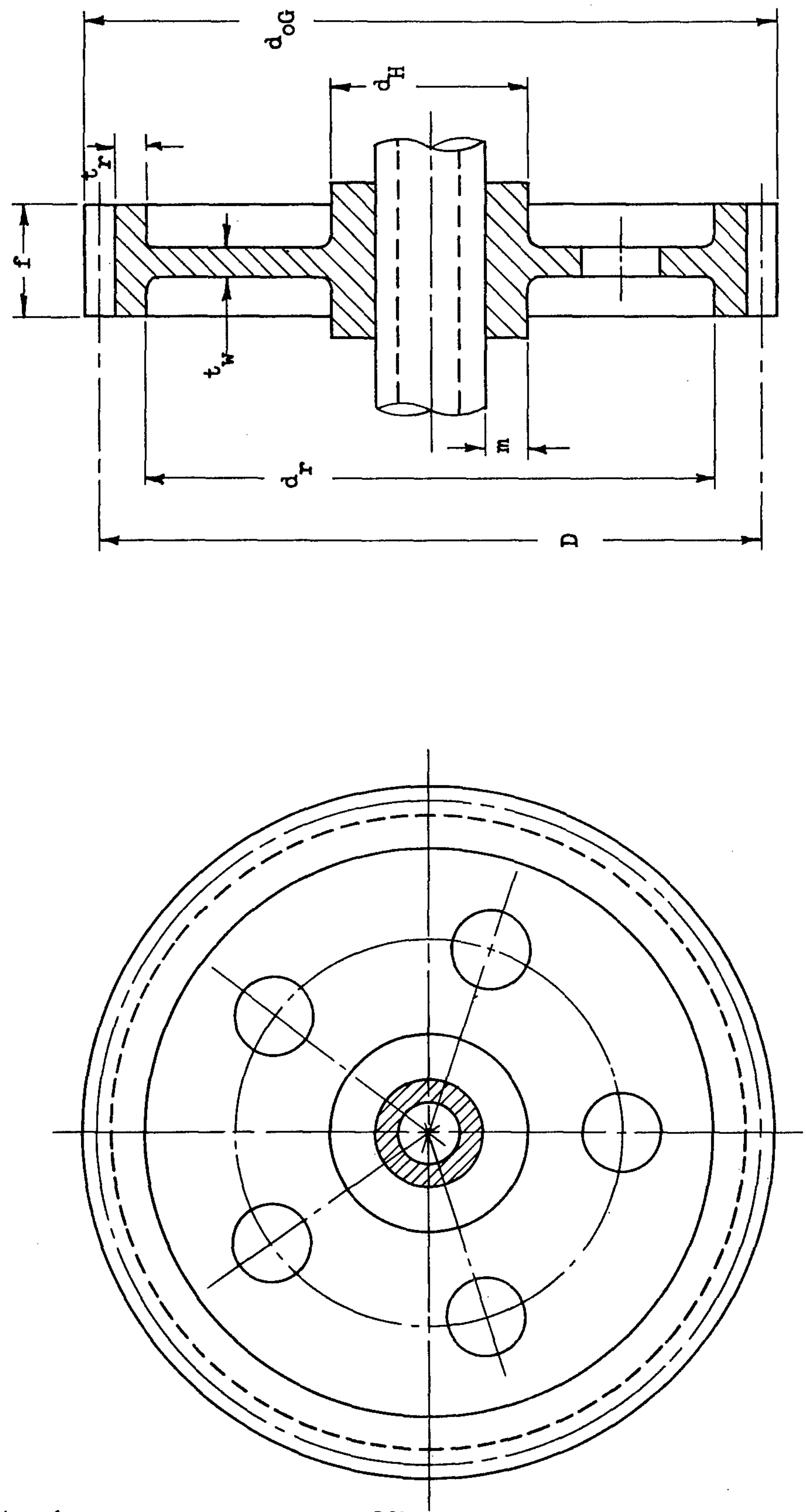



Fig. VI-5 NOMENCLATURE FOR TYPICAL GEAR

The minimum permissible thickness of metal above a keyway is found from

$$m = \frac{1}{P}\sqrt{\frac{n}{5}} \qquad \text{(VI-24)}$$

Good gear design dictates that the face width should be approximately 10/P.

Then, since the gear dedendum equals 1/P,

$$d_r = D - \frac{2}{P} - \frac{\pi f}{10} \qquad \text{(VI-25)}$$

or

$$d_r = D - (2 + \pi)\,\frac{f}{10} \qquad \text{(VI-26)}$$

$$t_w = \frac{\pi f}{20} + 0.125 \qquad \text{(VI-27)}$$

$$d_H = d_o + \frac{f}{5}\sqrt{\frac{n}{5}} \qquad \text{(VI-28)}$$

Substituting in Eq. (VI-21) gives:

$$a = \left[1 - \left(1 - \frac{f}{10}\,\frac{(2+\pi)}{D}\right)^2\right] + c\left(\frac{\pi}{20} + \frac{0.125}{f}\right)\left[\left(1 - \frac{f}{10}\,\frac{(2+\pi)}{D}\right)^2 - \left(\frac{d_o}{D} + \frac{f}{5D}\sqrt{\frac{n}{5}}\right)^2\right]$$

$$+ \left(\frac{d_o}{D} + \frac{f}{5D}\sqrt{\frac{n}{5}}\right)^2 \qquad \text{(VI-29)}$$

Examine the term $\frac{f}{5D}\sqrt{\frac{n}{5}}$ ,

$$\frac{f}{5D}\sqrt{\frac{n}{5}} = \sqrt{\frac{f^2\, n}{125\, D^2}}$$

Since,

$$P = \frac{n}{D}$$

and

$$P = \frac{10}{f}$$

Then,

$$\frac{f}{5D}\sqrt{\frac{n}{5}} = \frac{1}{3.54}\sqrt{\frac{f}{D}} \qquad \text{(VI-30)}$$

Substituting in Eq. (VI-29),

$$a = \left\{1 - \left[1 - \frac{(2+\pi)f}{10D}\right]^2\right\} + c\left(\frac{\pi}{20} + \frac{0.125}{f}\right)\left[\left(1 - \frac{(2+\pi)f}{10D}\right)^2 - \left(\frac{d_o}{D} + \frac{1}{3.54}\sqrt{\frac{f}{D}}\right)^2\right] + \left(\frac{d_o}{D} + \frac{1}{3.54}\sqrt{\frac{f}{D}}\right)^2 \qquad \text{(VI-31)}$$

Now let

$$X = 1 - \frac{(2+\pi)f}{10D} \qquad \text{(VI-32)}$$

$$B = c\left(\frac{\pi}{20} + \frac{0.125}{f}\right) \qquad \text{(VI-33)}$$

$$Z = \frac{d_o}{D} + \frac{1}{3.54}\sqrt{\frac{f}{D}} \qquad \text{(VI-34)}$$

Then,

$$a = 1 - X^2 + B\,(X^2 - Z^2) + Z^2 \qquad \text{(VI-35)}$$

Expanding, factoring, and regrouping gives

$$a = 1 - (1 - B)\,(X^2 - Z^2) \qquad \text{(VI-36)}$$

## G. Projected Areas for Gear Housing Weight

To calculate the weight of the gear case, it is necessary to determine the projected areas, $A_i$ and $A_o$, illustrated in Figs. VI-6 and VI-7.

In this derivation it is assumed that the outside diameters of the two gears are tangent and that the housing is tangent to both gears as shown in Fig. VI-8. It is further assumed that:

$$d_{oL} = R\,d_{op} \qquad \text{(VI-37)}$$

where:

$d_{oL}$ = outside diameter of the gear, in.

$d_{op}$ = outside diameter of the pinion, in.

$R$ = gear ratio

These assumptions will result in a slightly oversize gear case which will allow for clearance around the gears and an oil sump.

Referring to Fig. VI-8, it is seen that the projected area of the inner volume of the gear case is given by:

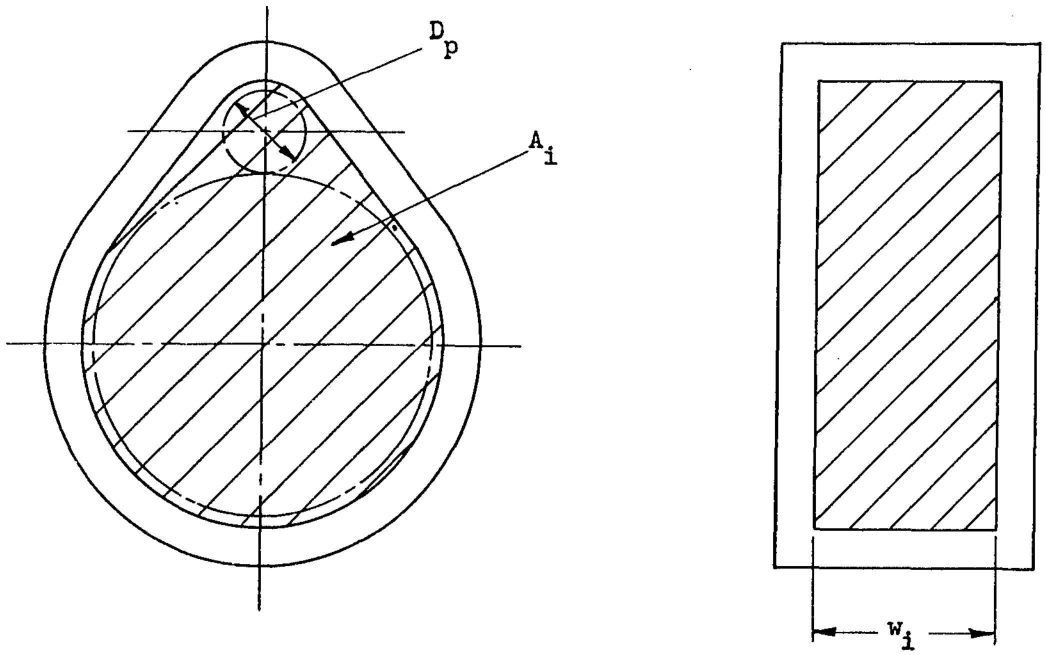



Fig. VI-6  MODIFIED GEAR CASE (INNER VOLUME)

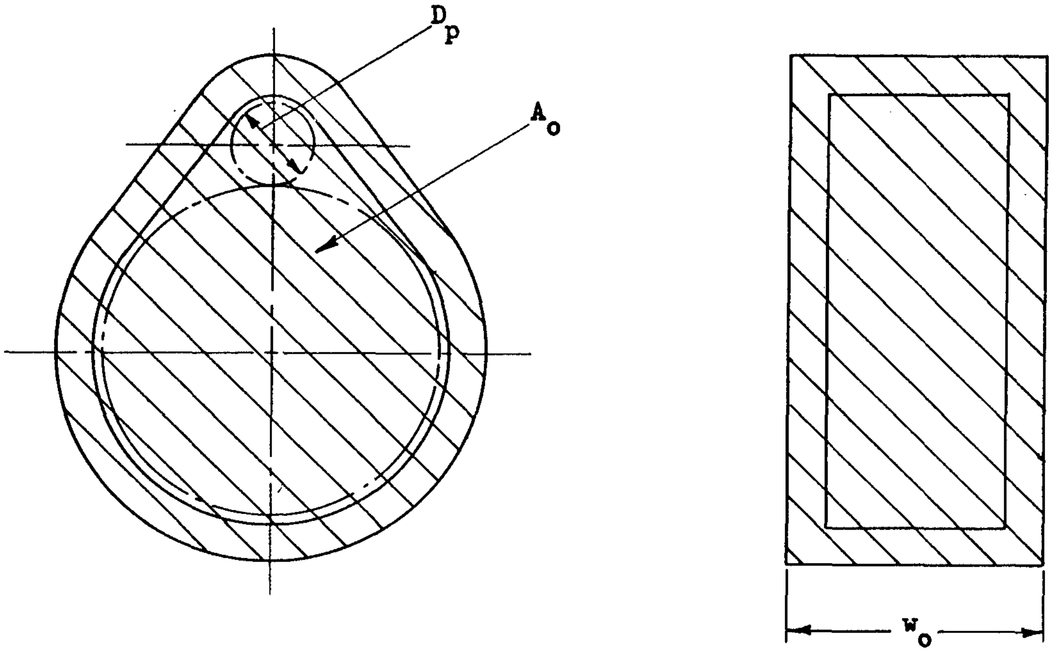
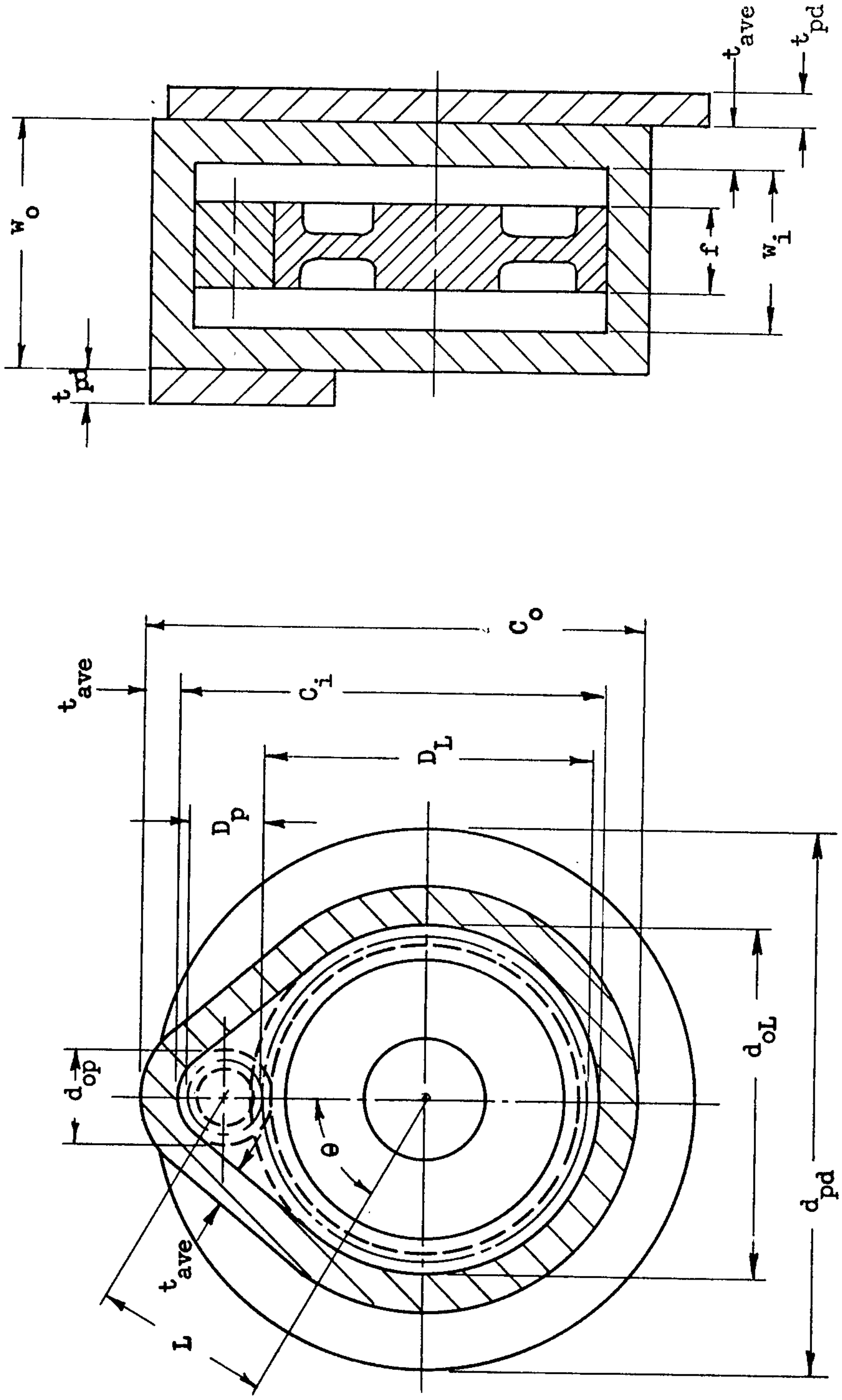



Fig. VI-7  MODIFIED GEAR CASE (OUTER VOLUME)




$w_i = f + K_w$

$w_o = f + K_w + 2\ t_{ave}$

Fig. VI-8 SINGLE-STEP SPEED REDUCER MODIFIED FOR WEIGHT ANALYSIS

$$A_i = \frac{\pi (d_{oL})^2}{4} \left(1 - \frac{\theta}{180}\right) + \frac{\pi (d_{op})^2}{4} \; \frac{\theta}{180} + L \, d_{op} + \frac{L(d_{oL} - d_{op})}{2} \qquad \text{(VI-38)}$$

where:

$L$ = length of the tangent line between gears, in.

$\theta$ = angle between a line through the gear centers and a line perpendicular to the common tangent of the outside diameters of the gears, degrees

From the geometry of the gear and gear housing:

$$L = d_{op}\sqrt{R} \qquad \text{(VI-39)}$$

$$\theta = \text{arc tan} \, \frac{2\sqrt{R}}{R - 1} \qquad \text{(VI-40)}$$

Then, Eq. (VI-38) may be written as:

$$A_i = d_{op}^{\;2} \left\{ \frac{\pi}{4} \left[ R^2 \left(1 - \frac{1}{180} \text{ arc tan } \frac{2\sqrt{R}}{R - 1}\right) + \frac{1}{180} \text{ arc tan } \frac{2\sqrt{R}}{R - 1} \right] + \frac{\sqrt{R}}{2} (1 + R) \right\} \qquad \text{(VI-41)}$$

Let:

$$F(R) = \frac{\pi}{4} \left[ R^2 \left(1 - \frac{1}{180} \text{ arc tan } \frac{2\sqrt{R}}{R - 1}\right) + \frac{1}{180} \text{ arc tan } \frac{2\sqrt{R}}{R - 1} \right] + \frac{\sqrt{R}}{2} (1 + R) \qquad \text{(VI-42)}$$

Equation (VI-42) is shown graphically in Fig. VI-9. Equation (VI-41) now becomes:

$$A_i = d_{op}^{\;2} \, F(R) \qquad \text{(VI-43)}$$

For 20° spur gears the addendum is equal to $\frac{1}{P}$. Then,

$$d_{op} = D_p \left(1 + \frac{2}{n_p}\right) \qquad \text{(VI-44)}$$

and

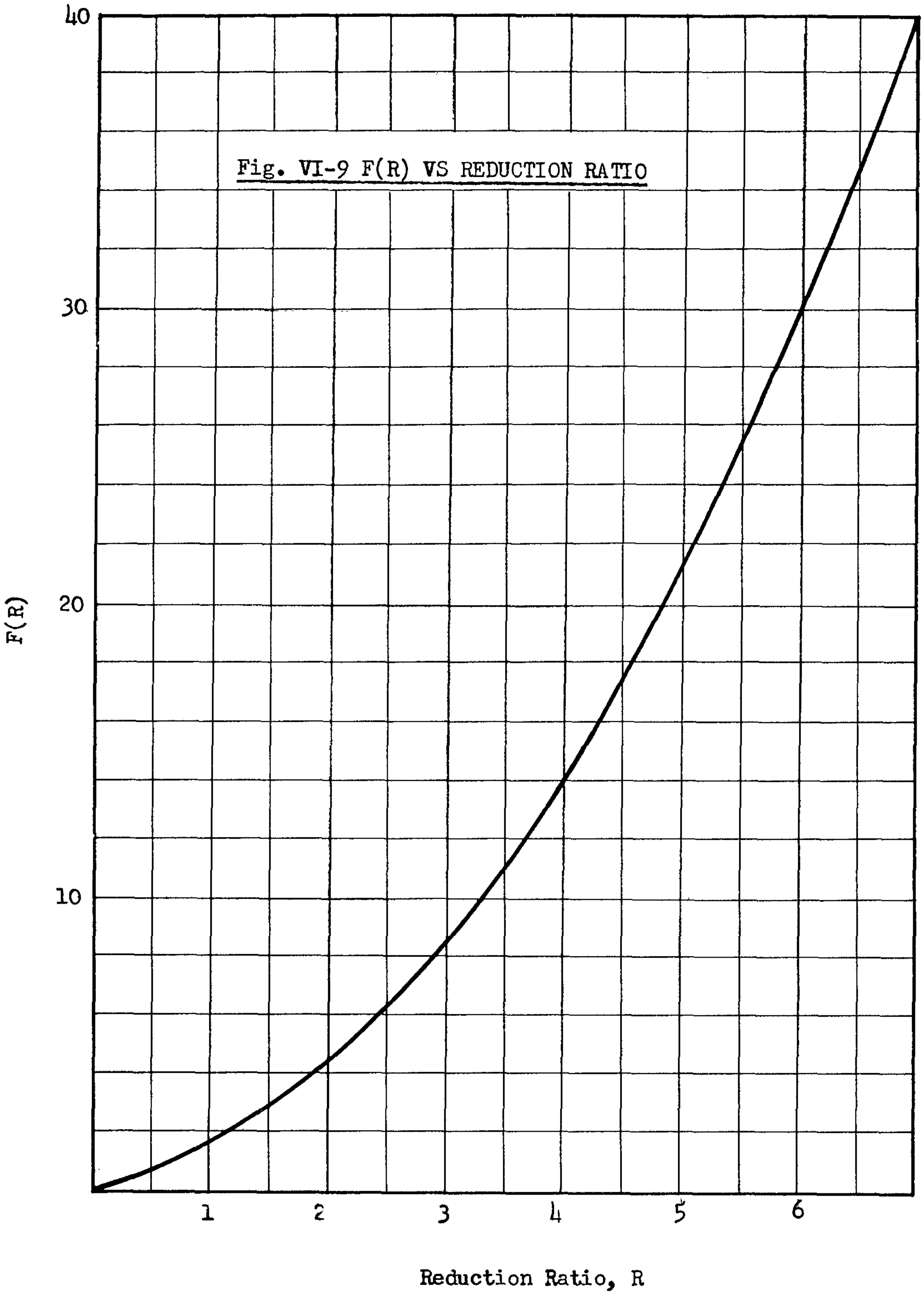



Fig. VI-9 F(R) VS REDUCTION RATIO

$$A_i = D_p^2 \, F(R) \, (1 + \frac{2}{n_p})^2 \qquad \text{(VI-45)}$$

Thus, the projected area of the inner volume of the gear housing has been determined in terms of a basic dimension of the gears, $D_p$.

It is now desired to determine the projected area of the outer volume of the gear case by multiplying the basic length dimension of the inner volume by some scale factor.

The inside height of the gear box, $C_i$, (See Fig. VI-8) is given by:

$$C_i = D_p \, (1 + R)(1 + \frac{2}{n_p}) \qquad \text{(VI-46)}$$

The outside height of the gear housing, $C_o$, (See Fig. VI-8) is given by:

$$C_o = D_p \, (1 + R)(1 + \frac{2}{n_p} + 2t_{ave}) \qquad \text{(VI-47)}$$

The scale factor, j, is given by:

$$j = \frac{C_o}{C_i} \qquad \text{(VI-48)}$$

$$j = 1 + \frac{2 \, t_{ave}}{D_p \, (1 + R) \, (1 + \frac{2}{n_p})} \qquad \text{(VI-49)}$$

Multiplying the basic length term in Eq. (VI-45) by the scale factor,

$$A_o = D_p^2 \, F(R) \left[ 1 + \frac{2 \, t_{ave}}{D_p \, (1 + R) \, (1 + \frac{2}{n_p})} \right]^2 (1 + \frac{2}{n_p})^2 \qquad \text{(VI-50)}$$

Expanding and factoring gives:

$$A_o = D_p^2 \, F(R) \left[ 1 + \frac{2}{n_p} + \frac{2t_{ave}}{D_p \, (1 + R)} \right]^2 \qquad \text{(VI-51)}$$

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VI-1. A. Vallance, and V. L. Doughtie, Design of Machine Members, McGraw Hill, 1943.

VI-2. V. L. Maleev, Machine Design, International Textbook Company, 1946.



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